PART I: PHYSICS

SECTION 1 (Maximum Marks: 21)

- This section contains **SEVEN** questions.

- Each question has **FOUR** options [A], [B], [C] and [D]. **ONLY ONE** of these four options is correct.

- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.

- For each question, marks will be awarded in one of the following categories:

  Full Marks : + 3 If only the bubble(s) corresponding to the correct option is darkened.
  Zero Marks : 0 If none of the bubbles is darkened.
  Negative Marks : – 1 In all other cases.

*Q.1 Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the instantaneous density ρ remains uniform throughout the volume. The rate of fractional change in density $\left(\dfrac{1}{\rho} \dfrac{d\rho}{dt} \right)$ is constant. The velocity v of any point on the surface of the expanding sphere is proportional to

[A] R   [B] R^3

[C] $\dfrac{1}{R}$ [D] $R^{2/3}$

Sol. **A**

$m = \rho \dfrac{4}{3} \pi R^3$

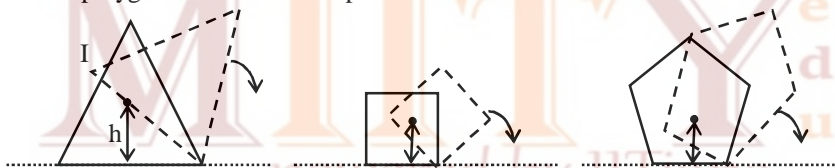$0 = \rho . 4\pi R^2 \dfrac{dR}{dt} + \dfrac{4}{3} \pi R^3 \dfrac{d\rho}{dt}$

$-\dfrac{1}{\rho} \dfrac{d\rho}{dt} = \dfrac{3}{R} \dfrac{dR}{dt}$

$-\dfrac{R}{3} \dfrac{1}{\rho} \dfrac{d\rho}{dt} = \dfrac{dR}{dt}$

$\dfrac{dR}{dt} \propto R$

$v \propto R$

*Q.2 Consider regular polygons with number of sides n = 3, 4, 5, ….. as shown in the figure. The center of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is Δ. Then Δ depends on n and h as

[A] $\Delta = h \sin^2 \dfrac{\pi}{n}$

[B] $\Delta = h \left(\dfrac{1}{\cos\left(\dfrac{\pi}{n}\right)} - 1 \right)$
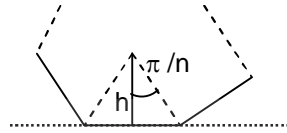
[C] $\Delta = h \sin \left(\dfrac{2\pi}{n} \right)$

[D] $\Delta = h \tan^2 \left(\dfrac{\pi}{2n} \right)$

Sol. **B**

For a regular polygon of n sides

$$\Delta = \frac{h}{\cos\dfrac{\pi}{n}} - h = h\left[\frac{1}{\cos\dfrac{\pi}{n}} - 1\right].$$

Q.3 A photoelectric material having work-function ϕ_0 is illuminated with light of wavelength λ $\left(\lambda < \dfrac{hc}{\phi_0}\right)$. The fastest photoelectron has a de Broglie wavelength λ_d. A change in wavelength of the incident light by $\Delta\lambda$ results in change $\Delta\lambda_d$ in λ_d. then the ratio $\Delta\lambda_d/\Delta\lambda$ is proportional to

[A] λ_d/λ

[B] λ_d^2/λ

[C] λ_d^3/λ

[D] λ_d^3/λ^2

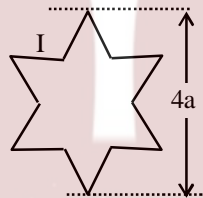*Sol.* **D**

$$\frac{P^2}{2m} = \frac{hc}{\lambda} - \phi_0$$

$$\frac{h^2}{2m.\lambda_D^2} = \frac{hc}{\lambda} - \phi_0$$

$$-\frac{h^2}{m}\frac{1}{\lambda_D^3}d\lambda_D = \frac{hc}{\lambda^2}d\lambda$$

$$\frac{\Delta\lambda_D}{\Delta\lambda} \propto \frac{\lambda_D^3}{\lambda^2}.$$

Q.4 A symmetric star conducting wire loop is carrying a steady state current I as shown in figure. The distance between the diametrically opposite vertices of the star is 4a. The magnitude of the magnetic field at the center of the loop is

[A] $\dfrac{\mu_0 I}{4\pi a}6[\sqrt{3}-1]$
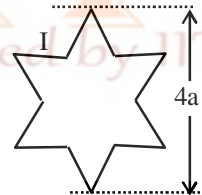
[B] $\dfrac{\mu_0 I}{4\pi a}6[\sqrt{3}+1]$

[C] $\dfrac{\mu_0 I}{4\pi a}3[\sqrt{3}-1]$
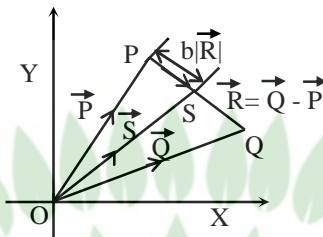
[D] $\dfrac{\mu_0 I}{4\pi a}3[2-\sqrt{3}]$

Sol. **A**

$$B = 12\left(\frac{\mu_0 I}{4\pi a}\right)(\sin 60 - \sin 30)$$

*Q.5 Three vectors $\vec{P}$, $\vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be any point on the vector $\vec{R}$. The distance between the point P ad S is $b|\vec{R}|$. The general relation among vectors $\vec{P}$, $\vec{Q}$ and $\vec{S}$ is

[A] $\vec{S} = (1-b)\vec{P} + b\vec{Q}$ [B] $\vec{S} = (b-1)\vec{P} + b\vec{Q}$

[C] $\vec{S} = (1-b^2)\vec{P} + b\vec{Q}$ [D] $\vec{S} = (1-b)\vec{P} + b^2\vec{Q}$

Sol. **A**

$\vec{S} - \vec{P} = b\vec{R}$

and $\vec{Q} - \vec{S} = (1-b)\vec{R}$ where $\vec{R} = \vec{Q} - \vec{P}$

From these two equations

$\vec{S} = (1-b)\vec{P} + b\vec{Q}$

*Q.6 A rocket is launched normal to the surface of earth, away from the Sun, along the line joining the Sun and the Earth. The Sun is 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of the Earth. The escape velocity from Earth's gravitational field is $v_e = 11.2$ km s^{-1}. The minimum initial (v_s) required for the rocket to be able to leave the Sun-earth system is closest to
(Ignore the rotation and revolution of the Earth and the presence of any other planet)
[A] $v_s = 22$ km s^{-1} [B] $v_s = 42$ km s^{-1}

[C] $v_s = 62$ km s^{-1} [D] $v_s = 72$ km s^{-1}

Sol. **B**

$$\frac{1}{2}mv_{es}^2 - \frac{GMm}{R} - \frac{G(3\times10^5 M)\times m}{2.5\times10^4 R} = 0$$

$\Rightarrow$ $V_{es} = \sqrt{13}$ $v_e = 40.3$ km/s

Q.7 A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is $\delta T = 0.01$ seconds and he measures the depth of the well to be L = 20 meters. Take the acceleration due to gravity g = 10 ms^{-2} and the velocity of sound is 300 ms^{-1}. Then the fractional error in the measurement, $\delta L/L$, is closest to
[A] 0.2 %   [B] 1 %

[C] 3 %   [D] 5 %

Sol. **B**

$$t = \sqrt{\frac{2L}{g}} + \frac{L}{C}$$

$$\frac{dt}{dL} = \sqrt{\frac{L}{g}} \times \frac{1}{2\sqrt{L}} + \frac{1}{C}$$

$$dL = \frac{dt}{\frac{1}{\sqrt{2gL}} + \frac{1}{C}}$$

$$\Rightarrow \quad \frac{dL}{L} \times 100 = \left(\frac{dt}{\frac{1}{\sqrt{2gL}} + \frac{1}{C}} \right) \frac{1}{L} \times 100$$

$$= \frac{15}{16\%} \approx 1\ \%$$

SECTION 2 (Maximum Marks: 28)

- This section contains **SEVEN** questions.

- Each question has **FOUR** options [A], [B], [C] and [D]. **ONE OR MORE THAN ONE** of these four options is(are) correct.

- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.

- For each question, marks will be awarded in one of the following categories:

Full Marks	: +4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
Partial Marks	: +1	For darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened.
Zero Marks	: 0	If none of the bubbles is darkened.
Negative Marks	: −2	In all other cases.

- For example, if [A], [C] and [D] are all the correct options for a question, darkening all these three will get +4 marks; darkening only [A] and [D] will get +2 marks; and darkening [A] and [B] will get −2 marks, as a wrong option is also darkened.

Q.8 A uniform magnetic field B exists in the region between x = 0 and $x = \frac{3R}{2}$ (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge +Q and momentum p directed along x-axis enters region 2 from region 1 at point P_1 (y = −R). Which of the following option(s) is/are correct?
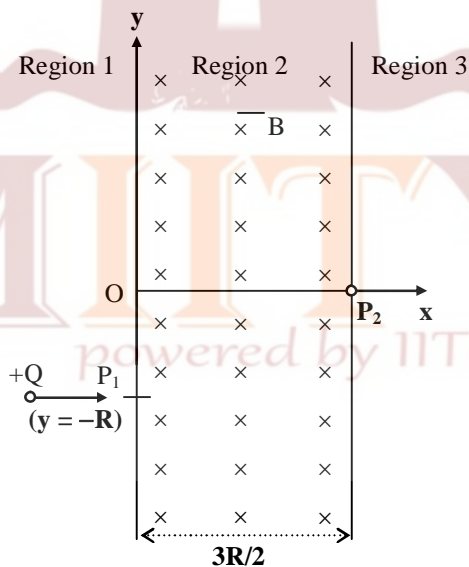
[A] For $B > \dfrac{2}{3}\dfrac{p}{QR}$, the particle will re-enter region 1

[B] For $B = \dfrac{8}{13}\dfrac{p}{QR}$, the particle will enter region 3 through the point P_2 on x-axis

[C] When the particle re-enters region 1 through the longest possible path in region 2, the magnitude of the change in its linear momentum between point P_1 and the farthest point from y-axis is $p/\sqrt{2}$

[D] For a fixed B, particles of same charge Q and same velocity v, the distance between the point P_1 and the point of re-entry into region 1 is inversely proportional to the mass of the particle

Sol.   **A, B**

$R' = \dfrac{p}{qB}$

$(R' - R)^2 + R^2 = R'^2$

$\Rightarrow \quad R' = \dfrac{13}{8}R$
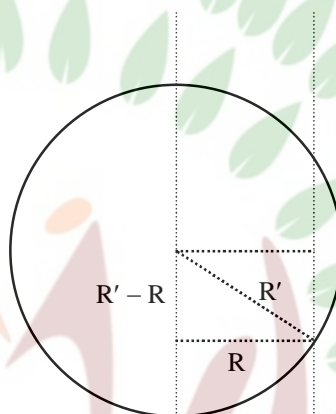
Q.9   The instantaneous voltages at three terminals marked X, Y and Z are given by

$V_x = V_0 \sin \omega t$

$V_y = V_0 \sin \left(\omega t + \dfrac{2\pi}{3} \right)$ and

$V_z = V_0 \sin \left(\omega t + \dfrac{4\pi}{3} \right).$

An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be

[A] $V_{XY}^{rms} = V_0 \sqrt{\dfrac{3}{2}}$
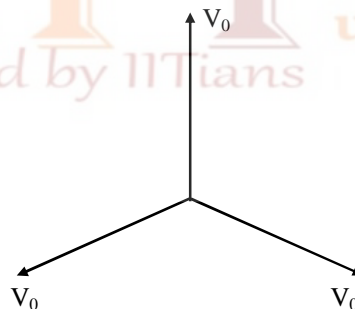
[B] $V_{YZ}^{rms} = V_0 \sqrt{\dfrac{1}{2}}$

[C] $V_{XY}^{rms} = V_0$
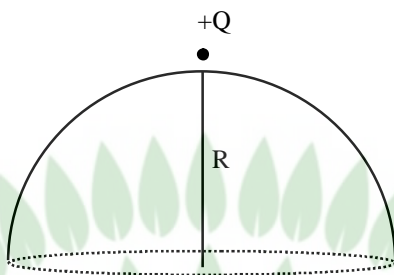
[D] independent of the choice of the two terminals

Sol.   **A, D**

$V_{XYO} = V_{Y2O} = V_{ZXO} = \sqrt{3}V_0$

$V_{XX}^{rms} = V_{YZ}^{rms} = \sqrt{\dfrac{3}{2}}V_0$

Q.10 A point charge +Q is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct?

+Q

R

[A] The electric flux passing through the *curved* surface of the hemisphere is $-\dfrac{Q}{2\varepsilon_0}\left(1-\dfrac{1}{\sqrt{2}}\right)$

[B] Total flux through the curved and the flat surfaces is $\dfrac{Q}{\varepsilon_0}$

[C] The component of the electric field normal to the flat surface is constant over the surface

[D] The circumference of the flat surface is an equipotential

Sol. **A, D**

Solid angle subtended by flat surface at position of charge

$= 2\pi(1 - \cos\theta)$

$= 2\pi(1 - \cos 45°)$

$= 2\pi\left(1 - \dfrac{1}{\sqrt{2}}\right)$

+Q

θ
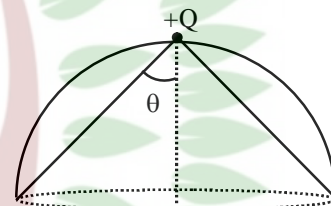
Flux entering through curved surface = flux leaving through flat surface

$\Rightarrow$ Flux entering through curved surface $= -\dfrac{Q}{\varepsilon_0}\times\dfrac{1}{4\pi}\times 2\pi\left(1-\dfrac{1}{\sqrt{2}}\right)$

$= -\dfrac{Q}{2\varepsilon_0}\left(1-\dfrac{1}{\sqrt{2}}\right)$

Also all points of circumference are at equal distance from the charge

Q.11 Two coherent monochromatic point sources S_1 and S_2 of wavelength $\lambda = 600$ nm are placed symmetrically on either side of the centre of the circle as shown. The sources are separated by a distance d = 1.8 mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct?

P_1

$\Delta\theta$

S_1 S_2 P_2

d
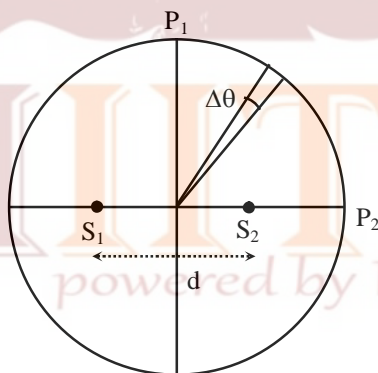
[A] A dark spot will be formed at the point P_2

[B] At P_2 the order of the fringe will be maximum

[C] The total number of fringes produced between P_1 and P_2 in the first quadrant is close to 3000

[D] The angular separation between two consecutive bright spots decreases as we move from P_1 to P_2 along the first quadrant

Sol. **B, C**
Path difference at point P
$\Delta x = d \cos \theta$ …(i)
At P_1, $\theta = 90° \Rightarrow \Delta x = 0 \rightarrow$ maxima
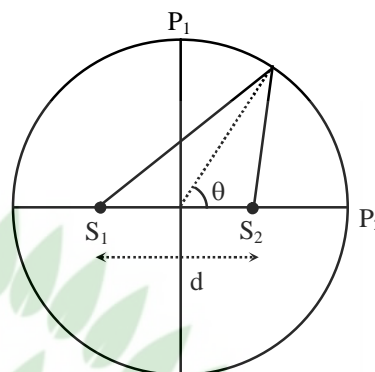At P_2, $\theta = 0 \Rightarrow \Delta x = d = n\lambda$

$\Rightarrow n = \dfrac{1.8 \times 10^{-3}}{600 \times 10^{-9}} = 3000 \rightarrow$ maxima
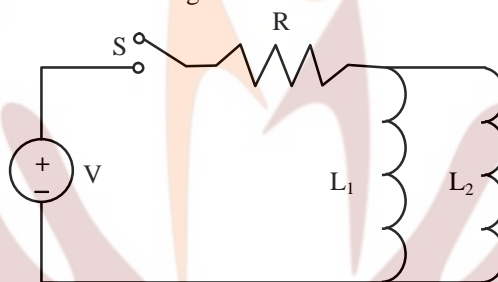
For maxima at P, $d \cos \theta = n\lambda$
or $d[-\sin \theta] \, d\theta = (dn)\lambda$

Angular fringe width, $\left| \dfrac{d\theta}{dn} \right| = \dfrac{\lambda}{d \sin \theta}$

as $\theta \uparrow d\theta \uparrow$

Q.12   A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At $t = 0$, the switch is closed and current begins to flow. Which of the following options is/are correct?

[A] After a long time, the current through L_1 will be $\dfrac{V}{R} \dfrac{L_2}{L_1 + L_2}$

[B] After a long time, the current through L_2 will be $\dfrac{V}{R} \dfrac{L_1}{L_1 + L_2}$

[C] The ratio of the currents through L_1 and L_2 is fixed at all times $(t > 0)$

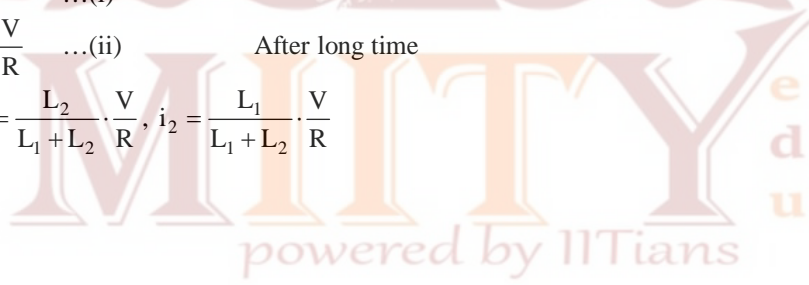[D] At $t = 0$, the current through the resistance R is $\dfrac{V}{R}$

Sol. **A, B, C**
Let current through R, L_1 and L_2 is i, i_1 and i_2 respectively
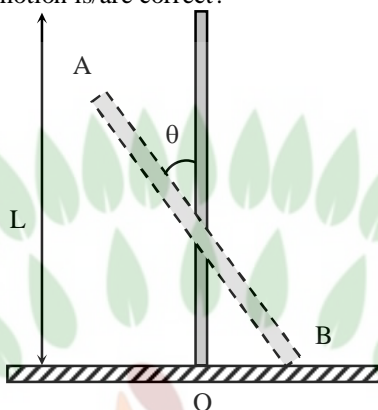$L_1 i_1 = L_2 i_2$ …(i)

$i_1 + i_2 = i = \dfrac{V}{R}$ …(ii)   After long time

Solving, $i_1 = \dfrac{L_2}{L_1 + L_2} \cdot \dfrac{V}{R}$, $i_2 = \dfrac{L_1}{L_1 + L_2} \cdot \dfrac{V}{R}$

*Q.13   A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vortical is θ. Which of the following statements about its motion is/are correct?


[A] The midpoint of the bar will fall vertically downward
[B] The trajectory of the point A is a parabola
[C] Instantaneous torque about the point in contact with the floor is proportional to sin θ
[D] When the bar makes an angle θ with the vertical, the displacement of its midpoint from the initial position is proportional to (1–cos θ)

Sol.   **A, C, D**
- There is no horizontal force on rod during its motion
⇒ C.M. will fall vertically downwards
- Net torque about point B(on the ground) = mgL sin θ
- Displacement mid point $= \dfrac{L}{2} - \dfrac{L}{2}\cos\theta$

*Q.14   A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct?
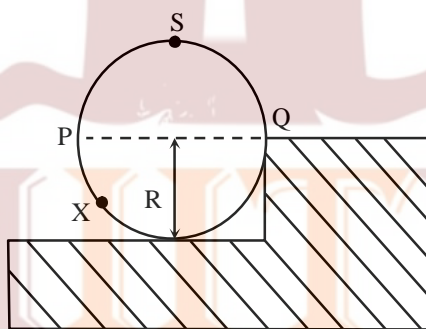

[A] If the force is applied at point P tangentially then τ decreases continuously as the wheel climbs
[B] If the force is applied normal to the circumference at point X then τ is constant
[C] If the force is applied normal to the circumference at point P then τ is zero
[D] If the force is applied tangentially at point S then τ ≠ 0 but the wheel never climbs the step

Sol.   **A**

- As the force acts as shown and considering its direction to be constant, then
$\tau_{net} = F(2R \cos \theta) - mgR \cos \theta = R \cos \theta(2F{-}mg)$
Which definitely infers that as θ increases, τ_{net} decreases.
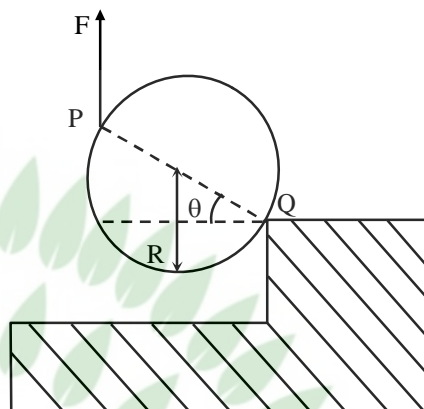
- As soon as force is applied as shown, it leads to frictional force between bottom point and horizontal surface, which will produce non-zero torque about point Q.
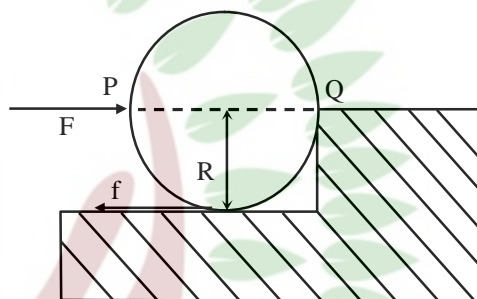
SECTION 3 (Maximum Marks : 12)

- This section contains **TWO** paragraphs

- Based on each table, there are **TWO** questions

- Each question has **FOUR** options [A], [B], [C], and [D]. **ONLY ONE** of these four options is correct

- For each question, darken the bubble corresponding to the correct option in the ORS

- For each question, marks will be awarded in <u>one of the following catogories</u>:

Full Marks   : +3 If only the bubble corresponding to the correct option is darkened
Zero Marks   :   0 In all other cases

PARAGRAPH 1

Consider a simple RC circuit as shown in Figure 1.

Process 1: In the circuit the switch S is closed at $t = 0$ and the capacitor is fully charged to voltage V_0 (i.e. charging continues for time $T \gg RC$). In the process some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_C.

Process 2: In a different process the voltage is first set to $\dfrac{V_0}{3}$ and maintained for a charging time $T \gg RC$. Then the voltage is raised to $\dfrac{2V_0}{3}$ without discharging the capacitor and again maintained for time $T \gg RC$. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final

voltage V_0 as in Process 1.

These two processes are depicted in Figure 2.

Figure 1   Figure 2
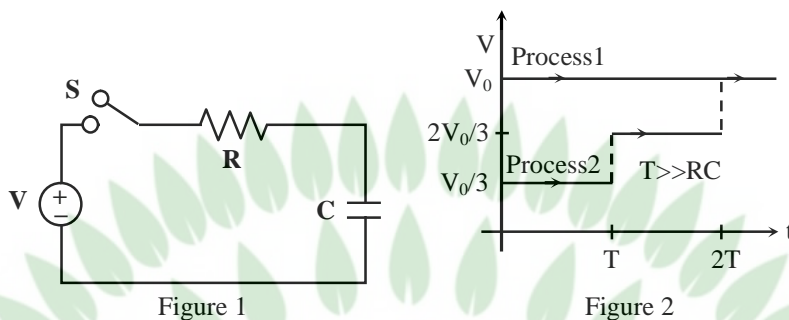
Q.15.   In Process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are released by:

[A] $E_C = E_D$

[B] $E_C = E_D \ln 2$

[C] $E_C = \dfrac{1}{2} E_D$

[D] $E_C = 2E_D$

Sol.   **A**

$$W_b = CV_0 \times V_0$$

$$\Delta U = \frac{1}{2}CV_0^2 = E_C$$

Heat dissipated $= W_b - E_C$ or $E_C = E_D = \dfrac{1}{2}CV_0^2$

Q.16   In Process 2, total energy dissipated across the resistance E_D is:

[A] $E_D = \dfrac{1}{2}CV_0^2$

[B] $E_D = 3\left(\dfrac{1}{2}CV_0^2\right)$

[C] $E_D = \dfrac{1}{3}\left(\dfrac{1}{2}CV_0^2\right)$

[D] $E_D = 3CV_0^2$

Sol.   **C**

If capacitor is charged from V_i to V_f; heat dissipated is

$$H = W_{battery} - \Delta U$$

$$= C(V_f - V_i)V_f - \left[\frac{1}{2}C\left(V_f^2 - V_i^2\right)\right]$$

$$= \frac{1}{2}C(V_f - V_i)^2$$

Total heat dissipated

$$= \frac{1}{2}C\left\{\left(\frac{V_0}{3} - 0\right)^2 + \left(\frac{2V_0}{3} - \frac{V_0}{3}\right)^2 + \left(V_0 - \frac{2V_0}{3}\right)^2\right\}$$

$$= \frac{1}{6}CV_0^2$$

PARAGRAPH 2

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating ring *rolls without slipping* on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g.
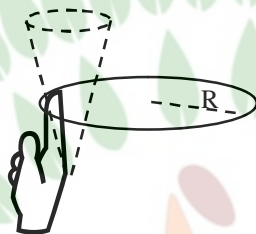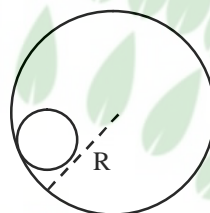
Figure 1   Figure 2

*Q.17   The total kinetic energy of the ring is

[A] $M\omega_0^2 R^2$

[B] $\dfrac{1}{2}M\omega_0^2 (R-r)^2$

[C] $M\omega_0^2 (R-r)^2$

[D] $\dfrac{3}{2}M\omega_0^2 (R-r)^2$

Sol.   **C**

*Q.18   The minimum value of ω_0 below which the ring will drop down is

[A] $\sqrt{\dfrac{g}{\mu(R-r)}}$

[B] $\sqrt{\dfrac{2g}{\mu(R-r)}}$

[C] $\sqrt{\dfrac{3g}{2\mu(R-r)}}$

[D] $\sqrt{\dfrac{g}{2\mu(R-r)}}$

Sol.   **A**

PART II: CHEMISTRY

SECTION 1 [Maximum Marks: 21]
- This section contains **SEVEN** questions
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories:
  Full Marks   :   +3 If only the bubble corresponding to the correct option is darkened
  Zero Marks   :   0 If none of the bubbles is darkened
  Negative Marks :   –1 In all other cases

Q.19. Pure water freezes at 273 K and 1 bar. The addition of 34.5 g of ethanol to 500 g of water changes the freezing point of the solution. Use the freezing point depression constant of water as 2 K kg mol⁻¹. The figures shown below represent plots of vapour pressure (V.P.) versus temperature (T). [molecular weight of ethanol is 46 g mol⁻¹]

Among the following, the option representing change the freezing point is
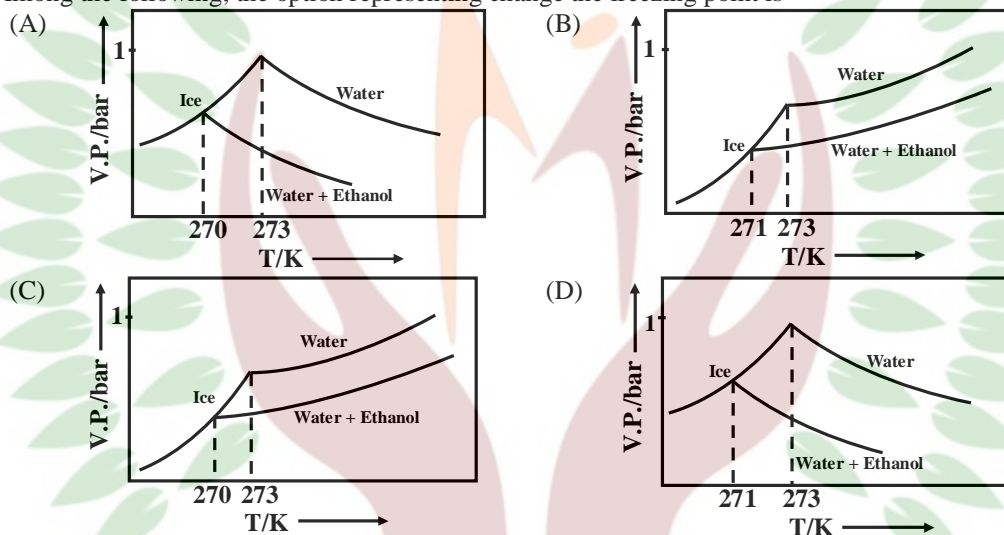
Sol. **C**

$$\Delta T_f = K_f \times m$$

$$= 2 \times \frac{34.5 \times 2}{46}$$

$$= 2 \times 1.5$$

$$= 3$$

Freezing point of ethanol + water mixture = 273- 3 = 270

Q.20. For the following cell,

$$Zn(s)\,|\,ZnSO_4(aq)\,||\,CuSO_4(aq)\,|\,Cu(s)$$

When the concentration of Zn^{2+} is 10 times the concentration of Cu^{2+}, the expression for ΔG (in J mol⁻¹) is [F is Faraday constant; R is gas constant; T is temperature; $E^0(cell) = 1.1$ V]

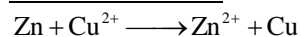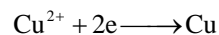[A] 1.1 F   [B] 2.303 RT – 2.2 F

[C] 2.303 RT + 1.1 F   [D] −2.2 F

Sol. **B**

$$Zn - 2e \longrightarrow Zn^{2+}$$

$$Cu^{2+} + 2e \longrightarrow Cu$$

$$\overline{Zn + Cu^{2+} \longrightarrow Zn^{2+} + Cu}$$

$$E_{cell} = E_{cell}^o - \frac{2.303RT}{nF} \log \frac{\left[Zn^{2+} \right]}{\left[Cu^{2+} \right]}$$

$$E_{cell} = 1.1 - \frac{2.303RT}{2F} \log 10$$

$$= 1.1 - \frac{2.303RT}{2F}$$

$$\Delta G = -nFE_{cell}$$

$$= -2F \left(1.1 - \frac{2.303RT}{2F} \right)$$

$$= 2.303RT - 2.2 \, F$$

*Q.21. The standard state Gibbs free energies of formation of C(graphite) and C(diamond) at T = 298 K are

$$\Delta_f G^0 \left[C(graphite) \right] = 0 \text{ kJmol}^{-1}$$

$$\Delta_f G^0 \left[C(diamond) \right] = 2.9 \text{ kJmol}^{-1}$$

The standard state means that the pressure should be 1 bar, and substance should be pure at a given temperature. The conversion of graphite [C(graphite)] to diamond [C(diamond)] reduces its volume by 2×10^{-6} m^3 mol^{-1}. If C(graphite) is converted to C(diamond) isothermally at T = 298 K, the pressure at which C(graphite) is in equilibrium with C(diamond), is

[Useful information: 1 J = 1 kg m^{2}s^{-2}; 1 Pa = 1 kg m^{-1}s^{-2}; 1 bar = 10^5 Pa]

[A] 14501 bar   [B] 58001 bar
[C] 1450 bar   [D] 29001 bar

Sol.   A

$$\Delta G = PdV$$

$$\left[\Delta_f G^o_{(diamond)} - \Delta_f G^o_{(graphite)} \right] = PdV$$

$$2.9 \times 10^3 \text{ J mol}^{-1} = P \times 2 \times 10^{-6} \text{ m}^3 \text{ mol}^{-1}$$

$$P = 1.45 \times 10^9 \text{ Pa}$$

$$P = 1.45 \times 10^9 \times 10^{-5} \text{ bar}$$

$$P = 1.45 \times 10^4 \text{ bar}$$

$$P = 14500 \text{ bar}$$

Q.22.   Which of the following combination will produce H$_2$ gas?
[A] Fe metal and conc. HNO$_3$ [B] Cu metal and conc. HNO$_3$
[C] Zn metal and NaOH(aq)   [D] Au metal and NaCN (aq) in the presence of air

Sol.   C

$$Zn + 2NaOH \longrightarrow Na_2ZnO_2 + H_2 \uparrow$$

*Q.23. The order of the oxidation state of the phosphorus atom in H$_3$PO$_2$, H$_3$PO$_4$, H$_3$PO$_3$, and H$_4$P$_2$O$_6$ is
[A] H$_3$PO$_3$ > H$_3$PO$_2$ > H$_3$PO$_4$> H$_4$P$_2$O$_6$ [B] H$_3$PO$_4$ > H$_3$PO$_2$ > H$_3$PO$_3$> H$_4$P$_2$O$_6$
[C] H$_3$PO$_4$ > H$_4$P$_2$O$_6$ > H$_3$PO$_3$> H$_3$PO$_2$ [D] H$_3$PO$_2$ > H$_3$PO$_3$ > H$_4$P$_2$O$_6$> H$_3$PO$_4$

Sol.   C
Species   Oxidation state of P
H$_3$PO$_4$ +5
H$_4$P$_2$O$_6$ +4
H$_3$PO$_3$ +3
H$_3$PO$_2$ +1

Q.24. The major product of the following reaction is

i) NaNO$_2$,HCl, 0^{0}C
ii) aq. NaOH

[A]

[B]

[C]

[D]

Sol. **C**

i) NaNO$_2$ – HCl
ii) 0°C

Aq. NaOH
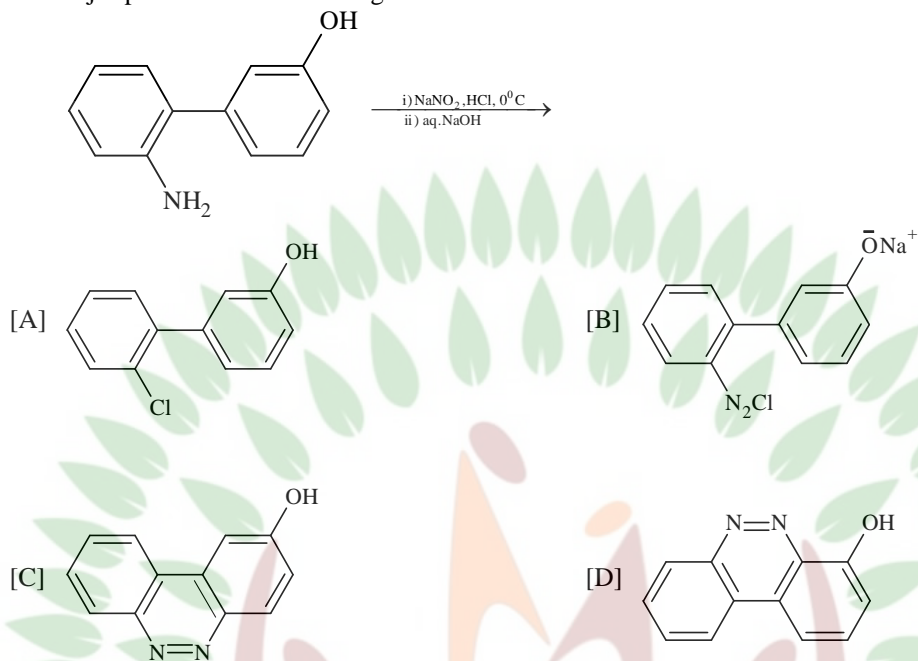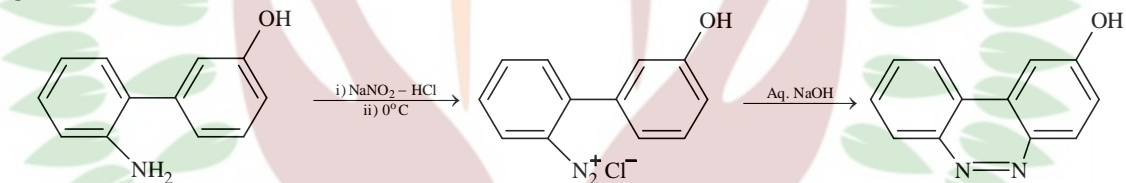
*Q.25. The order of basicity among the following compound is

I   II   III   IV

[A] II > I > IV > III   [B] IV > II > III > I
[C] IV > I > II > III   [D] I > IV > III > II

Sol. **C**

Conjugate acid has three equivalent canonical forms   Conjugate acid has two equivalent canonical forms   Conjugate acid has two equivalent canonical forms

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SECTION 2 [Maximum Marks: 28]
- This section contains **SEVEN** questions
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in *one of the following categories*:

  Full Marks   : +4   If only the bubble(s) corresponding to all the correct option(s) is (are) darkened

  Partial Marks   : +1   For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened.

  Zero Marks   : 0   If none of the bubbles is darkened

  Negative Marks : –2   In all other cases
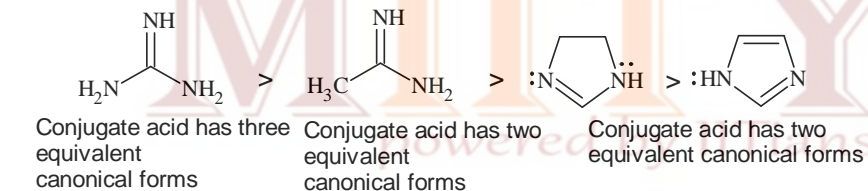- For example, if [A], [C] and [D] are all the correct options for a question, darkening all these three will get +4 marks; darkening only [A] and [D] will result in +2 marks; and darkening [A] and [B] will result in –2 marks, as a wrong option is also darkened.

Q.26. The correct statement(s) about surface properties is(are)

[A] Adsorption is accompanied by decrease in enthalpy and decrease in entropy of the system

[B] The critical temperatures of ethane and nitrogen are 563 K and 126 K, respectively. The adsorption of ethane will be more than that of nitrogen on same amount of activated charcoal at a given temperature

[C] Cloud is an emulsion type of colloid in which liquid is dispersed phase and gas is dispersion medium

[D] Brownian motion of colloidal particles does not depend on the size of the particles but depends on viscosity of the solution

Sol.   **A, B**

In adsorption process both $\Delta H \,\&\, \Delta S$ is – ve. Higher the critical temperature of a gas higher the extent of adsorption.

Cloud is not an emulsion.

Brownian motion depends on the size of the particles.

*Q.27. For a reaction taking place in a container in equilibrium with its surroundings, the effect of temperature on its equilibrium constant K in terms of change in entropy is described by

[A] With increase in temperature, the value of K for exothermic reaction decreases because entropy change of the system is positive

[B] With increase in temperature, the value of K for endothermic reaction increases because unfavourable change in entropy of the surroundings decreases

[C] With increase in temperature, the value of K for endothermic reaction increases because the entropy change of the system is negative

[D] With increase in temperature, the value of K for exothermic reaction decreases because favourable change in entropy of the surrounding decreases

Sol.   **B, D**

[B] With increase in temperature, the value of K for endothermic reaction increases because unfavourable change in entropy of the surroundings decreases

[D] With increase in temperature, the value of K for exothermic reaction decreases because favourable change in entropy of the surrounding decreases

Q.28. In a bimolecular reaction, the steric factor P was experimentally determined to be 4.5. The correct option(s) among the following is(are)

[A] The activation energy of the reaction is unaffected by the value of the steric factor

[B] Experimentally determined value of frequency factor is higher than that predicted by Arrhenius equation

[C] Since P = 4.5, the reaction will not proceed unless an effective catalyst is used

[D] The value of frequency factor predicted by Arrhenius equation is higher than that determined experimentally

Sol. **A, B**

[A] The activation energy of the reaction is unaffected by the value of the steric factor

[B] Experimentally determined value of frequency factor is higher than that predicted by Arrhenius equation

Q.29. For the following compounds, the correct statement(s) with respect to nucleophilic substitution reactions is(are)
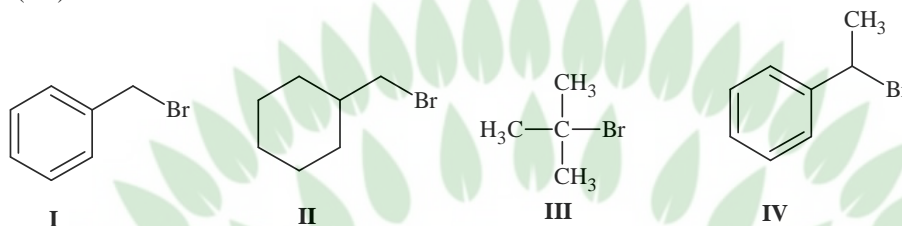
I   II   III   IV

[A] **I** and **III** follow S_N1 mechanism

[B] **I** and **II** follow S_N2 mechanism

[C] Compound **IV** undergoes inversion of configuration

[D] The order of reactivity for **I**, **III** and **IV** is: **IV > I > III**

Sol. **A, B, C, D**

Benzylic and 3º halides both follow S_N1 mechanism.

Benzylic and 1º halides both follow S_N2 mechanism.

Benzylic 2º halides can undergo inversion of configuration.

The order of reactivity would be IV > I > III if both S_N1 and S_N2 are considered suitably for substrates.

*Q.30. Among the following, the correct statement(s) is(are)

[A] $Al(CH_3)_3$ has the three-centre two-electron bonds in its dimeric structure

[B] BH_3 has the three-centre two-electron bonds in its dimeric structure

[C] $AlCl_3$ has the three-centre two-electron bonds in its dimeric structure

[D] The Lewis acidity of BCl_3 is greater than that of $AlCl_3$

Sol. **A, B, D**

Both $Al(CH_3)_3$ and BH_3 has 3c – 2e bonds in the dimeric structure.
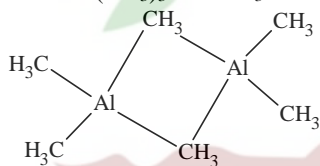
BCl_3 is stronger Lewis acid than $AlCl_3$.

Q.31. The option(s) with only amphoteric oxides is(are)

[A] Cr_2O_3, BeO, SnO, SnO_2 [B] Cr_2O_3, CrO, SnO, PbO

[C] NO, B_2O_3, PbO, SnO_2 [D] ZnO, Al_2O_3, PbO, PbO_2

Sol. **A, D**

Amphoteric oxides are

ZnO, Al_2O_3, PbO, PbO_2, BeO, SnO, SnO_2, Cr_2O_3

NO is a neutral oxide.

CrO is a basic oxide.

B_2O_3 is an acidic oxide

Q.32. Compounds **P** and **R** upon ozonolysis produce **Q** and **S**, respectively. The molecular formula of **Q** and **S** is C_8H_8O. **Q** undergoes Cannizzaro reaction but not haloform reaction, whereas **S** undergoes haloform reaction but not Cannizzaro reaction.

(i) $\mathbf{P} \xrightarrow[\text{ii) Zn/H}_2\text{O}]{\text{i) O}_3/\text{CH}_2\text{Cl}_2} \underset{(C_8H_8O)}{\mathbf{Q}}$

(ii) $\mathbf{R} \xrightarrow[\text{ii) Zn/H}_2\text{O}]{\text{i) O}_3/\text{CH}_2\text{Cl}_2} \underset{(C_8H_8O)}{\mathbf{S}}$
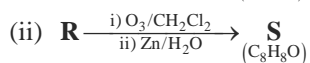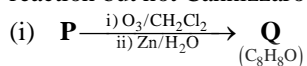
The option(s) with suitable combination of **P** and **R**, respectively, is(are)

[A]

[B]

[C]

[D]
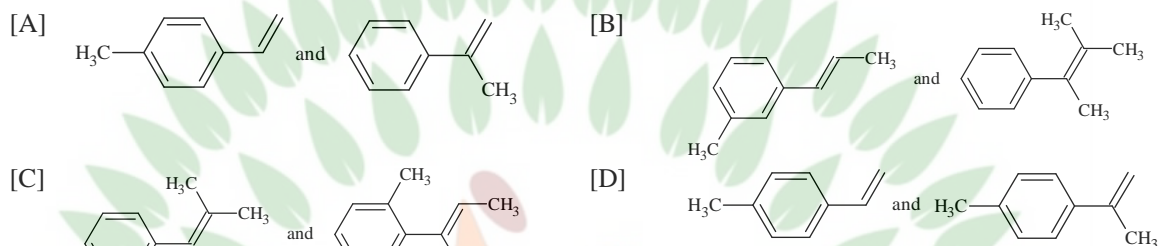
Sol. **A, B**
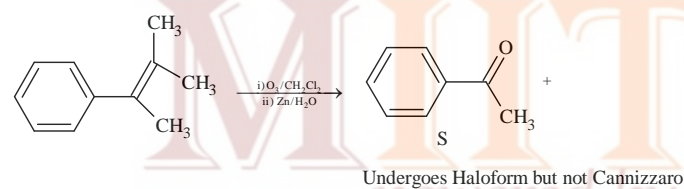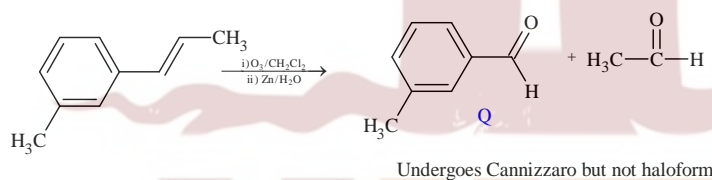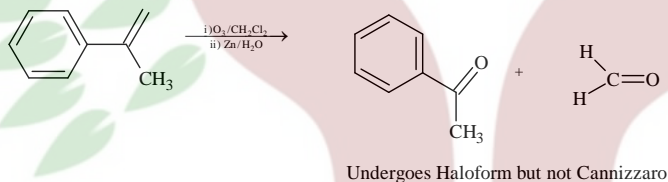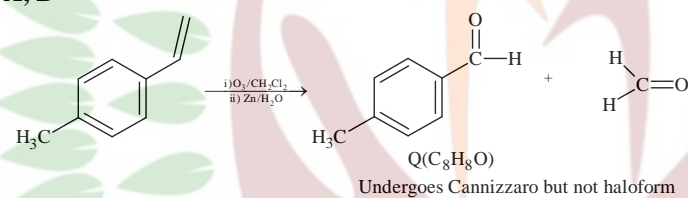
For (C) and (D) options no. of carbons are not matching

PARAGRAPH 1

Upon heating $KClO_3$ in the presence of catalytic amount of MnO_2, a gas **W** is formed. Excess amount of **W** reacts with white phosphorus to give **X**. The reaction of **X** with HNO_3 gives **Y** and **Z.**

Q.33   **W** and **X** are, respectively
       [A] O_3 and P_4O_6 [B] O_2 and P_4O_6
       [C] O_2 and P_4O_{10} [D] O_3 and P_4O_{10}

Q.34   **Y** and **Z** are, respectively
       [A] N_2O_3 and H_3PO_4 [B] N_2O_5 and HPO_3
       [C] N_2O_4 and HPO_3 [D] N_2O_4 and H_3PO_3

Solution for the Q. No. 33 to 34

$$2KClO_3 \xrightarrow{MnO_2} 2KCl + \underset{W}{3O_2}$$

$$P_4 + 5O_2 \longrightarrow \underset{X}{P_4O_{10}}$$

$$P_4O_{10} + 4HNO_3 \longrightarrow 4HPO_3 + 2N_2O_5$$
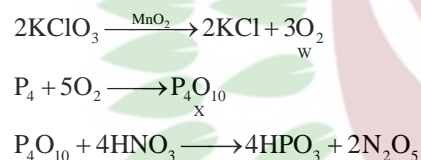
Q.33.   **C**

Q.34.   **B**

PARAGRAPH 2

The reaction of compound **P** with CH_3MgBr(excess) in $(C_2H_5)_2O$ followed by addition of H_2O gives **Q.** The compound **Q** on treatment with H_2SO_4 at 0^oC gives **R.** The reaction of **R** with CH_3COCl in the presence of anhydrous $AlCl_3$ in CH_2Cl_2 followed by treatment with H_2O produces compound **S** [Et in compound **P** is ethyl group]

Q. 35   The product **S** is

[A]

[B]

[C]

H_3COC

$(H_3C)_3C$ CH_3

H_3C CH_3

[D]

HO_3S

$(H_3C)_3C$ CH_3

O

$COCH_3$
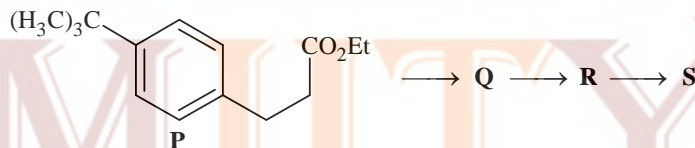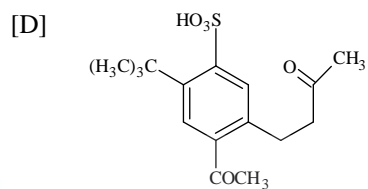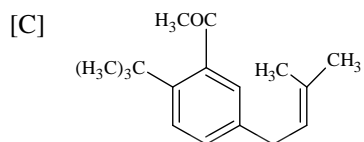
*Q. 36   The reactions, **Q** to **R** and **R** to **S**, are
    [A] Dehydration and Friedel-Crafts acylation
    [B] Aromatic sulfonation and Friedel-Crafts acylation
    [C] Friedel-Crafts alkylation, dehydration and Friedel-Crafts acylation
    [D] Friedel-Crafts alkylation and Friedel-Crafts acylation

Solution for the Q. No. 35 to 36

$(H_3C)_3C$ — [benzene ring] — CO_2Et

P

$\xrightarrow[\text{ii)}(C_2H_5)_2O, H_2O]{\text{i) }CH_3MgBr(\text{excess})}$

$(H_3C)_3C$ — [benzene ring] — C(OH)(CH_3) — CH_3

Q

$\xrightarrow[0^0C]{H_2SO_4}$

$(H_3C)_3C$ — [indane ring with two methyl groups]

R

Intramolecular Fridel-Craft alkylation & dehydration

$\xrightarrow[\text{ii)}H_2O]{\text{i)}CH_3COCl,\text{Anhydraws }AlCl_3,5CH_2Cl_2}$

$(H_3C)_3C$ — [indane ring with two methyl groups and C=O, CH_3]

$C=O$

CH_3

Fridel-Craft alkylation
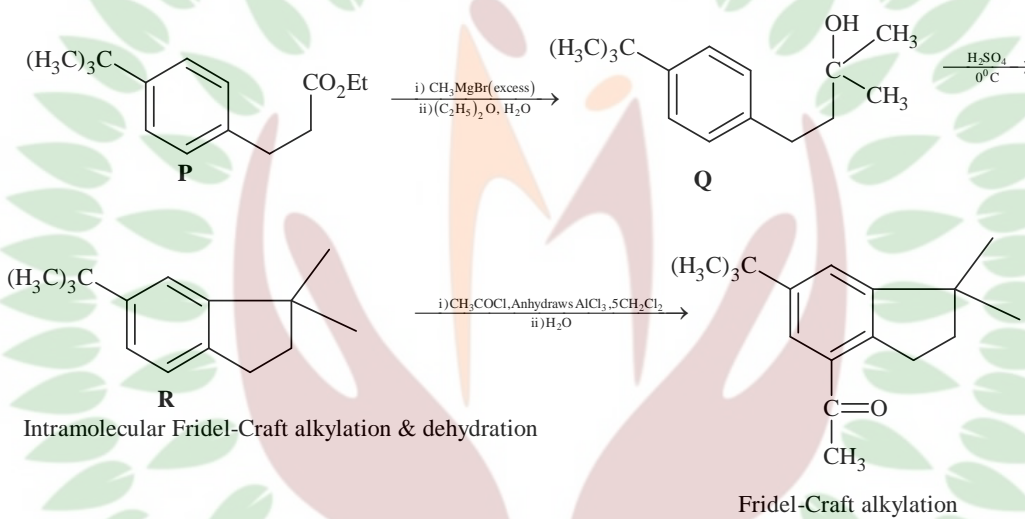
Q.35.   **A**

Q.36.   **C**

PART III: MATHEMATICS

SECTION 1 (Maximum Marks: 21)

- This section contains **SEVEN** questions.

- Each question has **FOUR** options [A], [B], [C] and [D]. **ONLY ONE** of these four options is correct.

- For each question, darken the bubble corresponding to the correct option in the ORS.

- For each question, marks will be awarded in one of the following categories:
  Full Marks   :   + 3   If only the bubble corresponding to the correct option is darkened.
  Zero Marks   :   0   If none of the bubbles is darkened.
  Negative Marks :   − 1   In all other cases.

Q.37 The equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$, is
[A] $14x + 2y - 15z = 1$ [B] $14x - 2y + 15z = 27$
[C] $14x + 2y + 15z = 31$ [D] $-14x + 2y + 15z = 3$

Sol. **C**
Let direction ratios of normal be $\vec{n}$

$$\Rightarrow \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 3 & -6 & -2 \end{vmatrix} = -14\hat{i} - 2\hat{j} - 15\hat{k}$$

$\Rightarrow$ equation of plane is
$-14(x - 1) - 2(y - 1) - 15(z - 1) = 0$
$\Rightarrow 14x + 2y + 15z = 31$

Q.38 Let O be the origin and let PQR be an arbitrary triangle. The point S is such that
$$\overrightarrow{OP} \cdot \overrightarrow{OQ} + \overrightarrow{OR} \cdot \overrightarrow{OS} = \overrightarrow{OR} \cdot \overrightarrow{OP} + \overrightarrow{OQ} \cdot \overrightarrow{OS} = \overrightarrow{OQ} \cdot \overrightarrow{OR} + \overrightarrow{OP} \cdot \overrightarrow{OS}$$
Then the triangle PQR has S as its
[A] centroid   [B] circumcentre
[C] incentre   [D] orthocenter

Sol. **D**
$\overrightarrow{OP} \cdot \overrightarrow{OQ} + \overrightarrow{OR} \cdot \overrightarrow{OS} = \overrightarrow{OR} \cdot \overrightarrow{OP} + \overrightarrow{OQ} \cdot \overrightarrow{OS}$
$\Rightarrow \overrightarrow{OP} \cdot \left(\overrightarrow{OQ} - \overrightarrow{OR} \right) = \overrightarrow{OS} \cdot \left(\overrightarrow{OQ} - \overrightarrow{OR} \right)$
$\Rightarrow \left(\overrightarrow{OP} - \overrightarrow{OS} \right) \cdot \left(\overrightarrow{RQ} \right) = 0$
$\left(\overrightarrow{SP} \right) \cdot \left(\overrightarrow{RQ} \right) = 0$
$\Rightarrow \overrightarrow{SP} \perp \overrightarrow{RQ}$
Similarly $\overrightarrow{SR} \perp \overrightarrow{QP}$ and $\overrightarrow{SQ} \perp \overrightarrow{PR}$
Hence, S is orthocentre

Q.39 If $y = y(x)$ satisfies the differential equation
$$8\sqrt{x} \left(\sqrt{9 + \sqrt{x}} \right) dy = \left(\sqrt{4 + \sqrt{9 + \sqrt{x}}} \right)^{-1} dx, \qquad x > 0$$
and $y(0) = \sqrt{7}$, then $y(256) =$
[A] 3   [B] 9
[C] 16   [D] 80

Sol. **A**

$$dy = \left(\cfrac{1}{8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)\left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)} \right) dx$$

Let $4+\sqrt{9+\sqrt{x}} = t$

$\Rightarrow \cfrac{1}{2\sqrt{9+\sqrt{x}}}\cdot\cfrac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow dy = \cfrac{dt}{2\sqrt{t}}$

$\Rightarrow 2dy = \cfrac{1}{\sqrt{t}} dt$

$2y = 2\sqrt{t}+c$

$\Rightarrow 2y = 2\sqrt{4+\sqrt{9+\sqrt{x}}} +c$

$y(0) = \sqrt{7} \Rightarrow c = 0$

$y = \sqrt{4+\sqrt{9+\sqrt{x}}}$

$y(256) = 3$

Q.40 If $f : R \to R$ is a twice differentiable function such that $f''(x) > 0$ for all $x \in R$, and $f\left(\dfrac{1}{2}\right) = \dfrac{1}{2}, f(1) = 1$,

then

[A] $f'(1) \le 0$

[B] $0 < f'(1) \le \dfrac{1}{2}$

[C] $\dfrac{1}{2} < f'(1) \le 1$

[D] $f'(1) > 1$

Sol. **D**

Let $h(x) = f(x) - x$

$h\left(\dfrac{1}{2}\right) = 0 = h(1)$

$\Rightarrow h'(\alpha) = 0$ for some $\alpha \in (0, 1)$ by rolle's theorem

$f'(\alpha) = 1$

as $f''(x) > 0 \Rightarrow f'(x)$ is increasing

$\therefore f'(1) > f'(\alpha)$

$f'(1) > 1$

Q.41 How many 3×3 matrices M with entries from {0, 1, 2} are there, for which the sum of the diagonal entries of M^TM is 5 ?

[A] 126

[B] 198

[C] 162

[D] 135

Sol. **B**

Let matrix $M = [k_{ij}]$

Then sum of diagonal entries $= \displaystyle\sum k_{ij}^2$

$\Rightarrow \displaystyle\sum k_{ij}^2 = 5$

where k_{ij} are from $\{0, 1, 2\}$

$\Rightarrow$ Total number of matrices $= {}^9C_5 + {}^9C_1 \cdot {}^8C_1 = 198$

Q.42 Let $S = \{1, 2, 3, \ldots, 9\}$. For $k = 1, 2, \ldots, 5$, let N_K be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5 =$

[A] 210 [B] 252

[C] 125 [D] 126

Sol. **D**

There are only 4 even numbers in S

$\therefore$ Any subset of 5 elements of S will have at least 1 odd number.

$\Rightarrow$ $N_1 + N_2 + N_3 + N_4 + N_5 = {}^9C_5 = 126$

Q.43 Three randomly chosen nonnegative integers x, y and z are found to satisfy the equation $x + y + z = 10$. Then the probability that z is even, is

[A] $\dfrac{36}{55}$ [B] $\dfrac{6}{11}$

[C] $\dfrac{1}{2}$ [D] $\dfrac{5}{11}$

Sol. **B**

Total number of solutions $= {}^{10+3-1}C_{3-1} = 66$

Favourable number of solutions $= {}^{11}C_1 + {}^9C_1 + {}^7C_1 + {}^5C_1 + {}^3C_1 + {}^1C_1 = 36$

$P(\text{req}) = \dfrac{36}{66} = \dfrac{6}{11}$

SECTION 2 (Maximum Marks: 28)

- This section contains **SEVEN** questions.

- Each question has **FOUR** options [A], [B], [C] and [D]. **ONE OR MORE THAN ONE** of these four options is(are) correct.

- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.

- For each question, marks will be awarded in <u>one of the following categories</u>:

  Full Marks : + 4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.

  Partial Marks : + 1 For darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened.

  Zero Marks : 0 If none of the bubbles is darkened.

  Negative Marks : – 2 In all other cases.

- For example, if [A], [C] and [D] are all the correct options for a question, darkening all these three will get +4 marks; darkening only [A] and [D] will get +2 marks; and darkening [A] and [B] will get –2 marks, as a wrong option is also darkened.

Q.44 If $g(x) = \displaystyle\int_{\sin x}^{\sin(2x)} \sin^{-1}(t)\, dt$, then

[A] $g'\left(\dfrac{\pi}{2}\right) = -2\pi$ [B] $g'\left(-\dfrac{\pi}{2}\right) = 2\pi$

[C] $g'\left(\dfrac{\pi}{2}\right) = 2\pi$ [D] $g'\left(-\dfrac{\pi}{2}\right) = -2\pi$

Sol. **Bonus**

$$g(x) = \int\limits_{\sin x}^{\sin(2x)} \sin^{-1}(t)\,dt$$

g'(x) = 2 (cos 2x) sin⁻¹ (sin 2x) – (cos x) sin⁻¹(sin x)

$$g'\left(\frac{\pi}{2}\right) = 2(-1)(0) - \cos\left(\frac{\pi}{2}\right)(1) = 0$$

$$g'\left(-\frac{\pi}{2}\right) = 0 \text{ no option is matching.}$$

* Q.45 Let α and β be nonzero real numbers such that $2(\cos\beta - \cos\alpha) + \cos\alpha \cos\beta = 1$. Then which of the following is/are true ?

[A] $\tan\left(\dfrac{\alpha}{2}\right) + \sqrt{3}\tan\left(\dfrac{\beta}{2}\right) = 0$

[B] $\sqrt{3}\tan\left(\dfrac{\alpha}{2}\right) + \tan\left(\dfrac{\beta}{2}\right) = 0$

[C] $\tan\left(\dfrac{\alpha}{2}\right) - \sqrt{3}\tan\left(\dfrac{\beta}{2}\right) = 0$

[D] $\sqrt{3}\tan\left(\dfrac{\alpha}{2}\right) - \tan\left(\dfrac{\beta}{2}\right) = 0$

Sol. **A, C**

cosβ (2 + cosα) = 1 + 2 cos α

$$\frac{\cos\beta}{1} = \frac{1+2\cos\alpha}{2+\cos\alpha}$$

$$\frac{\cos\beta+1}{\cos\beta-1} = \frac{3(\cos\alpha+1)}{\cos\alpha-1}$$

$$\frac{2\cos^2\dfrac{\beta}{2}}{-2\sin^2\dfrac{\beta}{2}} = \frac{3\times 2\cos^2\dfrac{\alpha}{2}}{-2\sin^2\dfrac{\alpha}{2}}$$

$$\tan^2\frac{\alpha}{2} = 3\tan^2\frac{\beta}{2}$$

$$\Rightarrow \quad \tan\frac{\alpha}{2} = \pm\sqrt{3}\tan\frac{\beta}{2}.$$

Q.46 If $f : R \to R$ is a differentiable function such that $f'(x) > 2f(x)$ for all $x \in R$, and $f(0) = 1$, then
[A] $f(x)$ is increasing in $(0, \infty)$
[B] $f(x)$ is decreasing in $(0, \infty)$
[C] $f(x) > e^{2x}$ in $(0, \infty)$
[D] $f'(x) < e^{2x}$ in $(0, \infty)$

Sol. **A, C**

f'(x) – 2f(x) > 0

$$\Rightarrow \frac{d}{dx}(f(x)\cdot e^{-2x}) > 0$$

$\Rightarrow$ f(x) . e⁻²ˣ is increasing function

$\Rightarrow$ f(x) . e⁻²ˣ > f(0) = 1 $\forall$ x > 0

f(x) > e²ˣ $\forall$ x > 0

$\Rightarrow$ f'(x) > 2f(x) > 2e²ˣ > 0 $\forall$ x > 0.

Q.47 Let $f(x) = \dfrac{1-x(1+|1-x|)}{|1-x|}\cos\left(\dfrac{1}{1-x}\right)$ for $x \neq 1$. Then

[A] $\lim\limits_{x\to1^-} f(x) = 0$

[B] $\lim\limits_{x\to1^-} f(x)$ does not exist

[C] $\lim\limits_{x\to1^+} f(x) = 0$

[D] $\lim\limits_{x\to1^+} f(x)$ does not exist

Sol. **A, D**

$$\lim_{x \to 1^-} f(x) = \lim_{h \to 0} \frac{h^2}{h} \cos \frac{1}{h} = 0$$

$$\lim_{x \to 1^+} f(x) = \lim_{h \to 0} \frac{-h(2+h)}{h} \cos \frac{1}{h} \text{ does not exist}$$

Q.48 If $f(x) = \begin{vmatrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$, then

[A] $f'(x) = 0$ at exactly three points in $(-\pi, \pi)$ [B] $f'(x) = 0$ at more than three points in $(-\pi, \pi)$
[C] $f(x)$ attains its maximum at $x = 0$ [D] $f(x)$ attains its minimum at $x = 0$

Sol. **B, C**

$$f(x) = \begin{vmatrix} \cos 2x & \cos 2x & \sin 2x \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = \cos 4x + \cos 2x$$

Now f'(x) = −2sin2x − 4sin4x = 0
$\Rightarrow$ f'(x) = 2sin2x (1 + 4 cos 2x) = 0

Then sin 2x = 0 or cos 2x = $-\dfrac{1}{4}$

For sin 2x = 0; x = 0, π/2 − π/2
For cos 2x = − 1/4 there are four solutions.
f'(x) = 0 has more than three solutions.
Again f''(x) = − (4 cos 2x + 16 cos 4x)
$\Rightarrow$ f''(0) < 0

Q.49 If the line $x = \alpha$ divides the area of region $R = \{(x, y) \in \mathbb{R}^2 : x^3 \le y \le x, 0 \le x \le 1\}$ into two equal parts, then

[A] $0 < \alpha \le \dfrac{1}{2}$ [B] $\dfrac{1}{2} < \alpha < 1$

[C] $2\alpha^4 - 4\alpha^2 + 1 = 0$ [D] $\alpha^4 + 4\alpha^2 - 1 = 0$
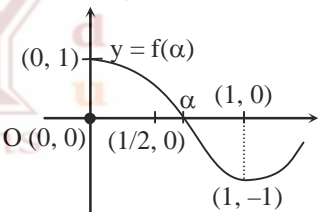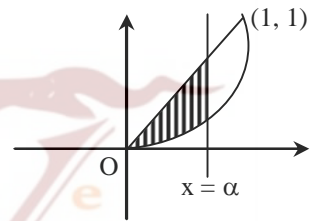
Sol. **B, C**

$$\frac{1}{2}\left(\int_0^1 (x - x^3)\,dx \right) = \int_0^\alpha (x - x^3)\,dx$$

$$\frac{1}{8} = \left| \frac{x^2}{2} - \frac{x^4}{4} \right|_0^\alpha$$

$$\frac{1}{2} = 2\alpha^2 - \alpha^4$$

$2\alpha^4 - 4\alpha^2 + 1 = 0$
$f(\alpha) = 2\alpha^4 - 4\alpha^2 + 1$

Q.50 If $I = \sum\limits_{k=1}^{98} \int\limits_{k}^{k+1} \dfrac{k+1}{x(x+1)} dx$, then

[A] $I > \log_e 99$ [B] $I < \log_e 99$

[C] $I < \dfrac{49}{50}$ [D] $I > \dfrac{49}{50}$

Sol. **B, D**

$$\sum\limits_{k=1}^{98} \int\limits_{k}^{k+1} \dfrac{1}{x+1} dx < \sum\limits_{k=1}^{98} \int\limits_{k}^{k+1} \dfrac{k+1}{x(x+1)} dx < \sum\limits_{k=1}^{98} \int\limits_{k}^{k+1} \dfrac{dx}{x}$$

$$\sum\limits_{k=1}^{98} \left(\ell n(k+2) - \ell n(k+1) \right) < I < \sum\limits_{k=1}^{98} \left(\ell n(k+1) - \ell nk \right)$$

$\ell n\ 50 < I < \ell n\ 99$

$\Rightarrow \quad \dfrac{49}{50} < I < \ell n99$.

SECTION 3 (Maximum Marks: 12)

- This section contains **TWO** paragraphs.

- Based on each paragraph, there are **TWO** questions.

- Each question has **FOUR** options [A], [B], [C] and [D]. **ONLY ONE** of these four options is correct.

- For each question, darken the bubble corresponding to the correct option in the ORS.

- For each question, marks will be awarded in <u>one of the following categories:</u>
  Full Marks   :   + 3   If only the bubble corresponding to the correct option is darkened.
  Zero Marks   :   0   In all other cases.

PARAGRAPH 1

Let O be the origin, and $\overrightarrow{OX}$, $\overrightarrow{OY}$, $\overrightarrow{OZ}$ be three unit vectors in the directions of the sides $\overrightarrow{QR}$, $\overrightarrow{RP}$, $\overrightarrow{PQ}$, respectively, of a triangle PQR.

Q.51 $\left| \overrightarrow{OX} \times \overrightarrow{OY} \right| =$
[A] $\sin(P + Q)$ [B] $\sin 2R$
[C] $\sin(P + R)$ [D] $\sin(Q + R)$

Sol. **A**
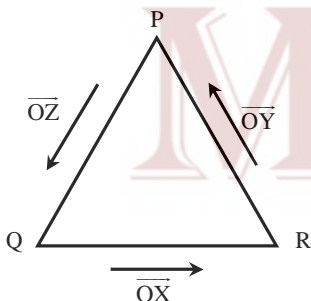
$$\left| \overrightarrow{OX} \times \overrightarrow{OY} \right| = \left| \overrightarrow{OX} \right| \left| \overrightarrow{OY} \right| \sin(\pi - R)$$

$$= \sin R = \sin(P + Q)$$

Q.52   If the triangle *PQR* varies, then the minimum value of

$$\cos(P+Q)+\cos(Q+R)+\cos(R+P)$$

is

[A] $-\dfrac{5}{3}$ [B] $-\dfrac{3}{2}$

[C] $\dfrac{3}{2}$ [D] $\dfrac{5}{3}$

Sol.   **B**

$$-(\cos P + \cos Q + \cos R) = \overrightarrow{OX}\cdot\overrightarrow{OY} + \overrightarrow{OY}\cdot\overrightarrow{OZ} + \overrightarrow{OZ}\cdot\overrightarrow{OX}$$

$$= \frac{\left(\overrightarrow{OX}+\overrightarrow{OY}+\overrightarrow{OZ}\right)^2 - \left(\left|\overrightarrow{OX}\right|^2 + \left|\overrightarrow{OY}\right|^2 + \left|\overrightarrow{OZ}\right|^2\right)}{2}$$

$$\geq -\frac{3}{2}$$

PARAGRAPH 2

Let p, q be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For $n = 0, 1, 2, \dots$, let $a_n = p\alpha^n + q\beta^n$.

FACT : If a and b are rational numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$.

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* Q.53 $a_{12} =$

[A] $a_{11} - a_{10}$ [B] $a_{11} + a_{10}$

[C] $2a_{11} + a_{10}$ [D] $a_{11} + 2a_{10}$

Sol.   **B**

As $a_{n+1} - a_n - a_{n-1} = 0$

So $a_{12} = a_{11} + a_{10}$

* Q.54   If $a_4 = 28$, then $p + 2q =$

[A] 21   [B] 14

[C] 7   [D] 12

Sol.   **D**

$a_4 = 2a_0 + 3a_1$

$a_4 = (q - p)(3\beta) + 5p + 2q = 28$

$\Rightarrow$ $p = q = 4$.