CODE **3**

PAPER 2

Time: 3 Hours **Maximum Marks: 180**

INSTRUCTIONS

A. General:

1. This booklet is your Question Paper. Do not break the seals of this booklet before being instructed to do so by the invigilators.
2. Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers and electronic gadgets are NOT allowed inside the examination hall.
3. Write your name and roll number in the space provided on the back cover of this booklet.
4. Answers to the questions and personal details are to be filled on a two-part carbon-less paper, which is provided separately. These parts should only be separated at the end of the examination when instructed by the invigilator. The upper sheet is a machine-gradable Objective Response Sheet (ORS) which will be retained by the invigilator. You will be allowed to take away the bottom sheet at the end of the examination.
5. **Using a black ball point pen darken the bubbles on the upper original sheet.** Apply sufficient pressure so that the impression is created on the bottom duplicate sheet.

B. Question Paper Format

6. The question paper consists of three parts (Physics, Chemistry and Mathematics). Each part consists of three sections.
   Section 1 contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE are correct**
7. **Section 2** contains **4 paragraphs** each describing theory, experiment, data etc. **Eight questions** relate to four paragraphs with two questions on each paragraph. Each question of a paragraph has **ONLY ONE correct answer** among the four choices (A), (B), (C) and (D).
8. **Section 3** contains **4 multiple choice questions** relate to four paragraphs with two questions on each paragraph. Each question of a paragraph has **ONLY ONE CORRECT ANSWER** among the four choices (A), (B), (C) and (D).

C. Marking Scheme

9. For each question in **Section 1,** you will be awarded **3 marks** if you darken all the bubble(s) corresponding to only the correct answer(s) and **zero mark** if no bubbles are darkened. In all other cases, **minus one (–1) mark** will be awarded.
10. For each question **Section 2 and 3**, you will be awarded **3 marks** if you darken the bubble corresponding to only the correct answer and **zero mark** if no bubbles are darkened. In all other cases, **minus one (–1) mark** will be awarded.

PART - I: PHYSICS

SECTION – 1 (One or more options correct Type)

This section contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** are correct.

*1. Two bodies, each of mass M, are kept fixed with a separation 2L. A particle of mass m is projected from the midpoint of the line joining their centres, perpendicular to the line. The gravitational constant is G. The correct statement(s) is (are)

(A) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\frac{GM}{L}}$

(B) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$.

(C) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$

(D) The energy of the mass m remains constant.

Sol. **(B)**

$$\frac{-2GMm}{L}+\frac{1}{2}mv^2=0$$

$$\Rightarrow v=2\sqrt{\frac{GM}{L}}$$

Note: *The energy of mass 'm' means its kinetic energy (KE) only and not the potential energy of interaction between m and the two bodies (of mass M each) – which is the potential energy of the system.*

*2. A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time t = 0 with an initial velocity u_0. When the speed of the particle is 0.5 u_0. It collides elastically with a rigid wall. After this collision,

(A) the speed of the particle when it returns to its equilibrium position is u_0.

(B) the time at which the particle passes through the equilibrium position for the first time is $t=\pi\sqrt{\frac{m}{k}}$.

(C) the time at which the maximum compression of the spring occurs is $t=\frac{4\pi}{3}\sqrt{\frac{m}{k}}$.

(D) the time at which the particle passes through the equilibrium position for the second time is $t=\frac{5\pi}{3}\sqrt{\frac{m}{k}}$.

Sol. **(A, D)**

$v=u_0\sin\omega t$ (suppose t_1 is the time of collision) $\frac{u_0}{2}=u_0\cos\omega t_1 \Rightarrow t_1=\frac{\pi}{3\omega}$

Now the particle returns to equilibrium position at time $t_2 = 2t_1$ i.e. $\frac{2\pi}{3\omega}$ with the same mechanical energy i.e. its speed will u_0 .

Let t_3 is the time at which the particle passes through the equilibrium position for the second time.

$$\therefore\ t_3 = \frac{T}{2} + 2t_1$$
$$= \frac{\pi}{\omega} + \frac{2\pi}{3\omega} = \frac{5\pi}{3\omega}$$
$$= \frac{5\pi}{3}\sqrt{\frac{m}{k}}$$

Energy of particle and spring remains conserved.

3. A steady current I flows along an infinitely long hollow cylindrical conductor of radius R. This cylinder is placed coaxially inside an infinite solenoid of radius 2R. The solenoid has n turns per unit length and carries a steady current I. Consider a point P at a distance r from the common axis. The correct statement(s) is (are)
(A) In the region $0 < r < R$, the magnetic field is non-zero
(B) In the region $R < r < 2R$, the magnetic field is along the common axis.
(C) In the region $R < r < 2R$, the magnetic field is tangential to the circle of radius r, centered on the axis.
(D) In the region $r > 2R$, the magnetic field is non-zero.

Sol. **(A, D)**
Due to field of solenoid is non zero in region $0 < r < R$ and non zero in region $r>2R$ due to conductor.

*4. Two vehicles, each moving with speed u on the same horizontal straight road, are approaching each other. Wind blows along the road with velocity w. One of these vehicles blows a whistle of frequency f_1. An observer in the other vehicle hears the frequency of the whistle to be f_2. The speed of sound in still air is V. The correct statement(s) is (are)
(A) If the wind blows from the observer to the source, $f_2 > f_1$.
(B) If the wind blows from the source to the observer, $f_2 > f_1$.
(C) If the wind blows from observer to the source, $f_2 < f_1$.
(D) If the wind blows from the source to the observer $f_2 < f_1$.

Sol. **(A, B)**
If wind blows from source to observer
$$f_2 = f_1\left(\frac{V+w+u}{V+w-u}\right)$$
When wind blows from observer towards source
$$f_2 = f_1\left(\frac{V-w+u}{V-w-u}\right)$$
In both cases, $f_2 > f_1$.

*5. Using the expression $2d\sin\theta = \lambda$, one calculates the values of d by measuring the corresponding angles θ in the range θ to 90°. The wavelength λ is exactly known and the error in θ is constant for all values of θ. As θ increases from 0°,
(A) the absolute error in d remains constant.
(B) the absolute error in d increases
(C) the fractional error in d remains constant.
(D) the fractional error in d decreases.

Sol. **(D)**
$$d = \frac{\lambda}{2\sin\theta}$$
$$\ln d = \ln\left(\frac{\lambda}{2}\right) - \ln\sin\theta$$

$$\frac{\Delta d}{d} = 0 - \frac{\cos\theta d\theta}{\sin\theta}$$

$$\left(\frac{\Delta d}{d}\right)_{max} = \pm\cot\theta\Delta\theta$$

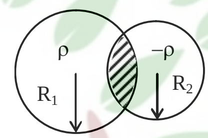
Also $(\Delta d)_{max} = d\cot\theta\Delta\theta$

$$\frac{\lambda}{2\sin\theta}\cot\theta\Delta\theta$$

$$= \frac{\lambda}{2}\frac{\cos\theta}{\sin^2\theta}\Delta\theta$$

As θ increases $\cot\theta$ decreases and $\frac{\cos\theta}{\sin^2\theta}$ also decreases.

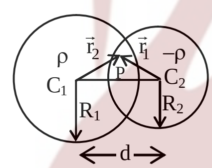
6. Two non-conducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region,



(A) the electrostatic field is zero
(B) the electrostatic potential is constant
(C) the electrostatic field is constant in magnitude
(D) the electrostatic field has same direction

Sol. **(C, D)**

In triangle PC_1C_2



$\vec{r}_2 = \vec{d} + \vec{r}_1$

The electrostatic field at point P is

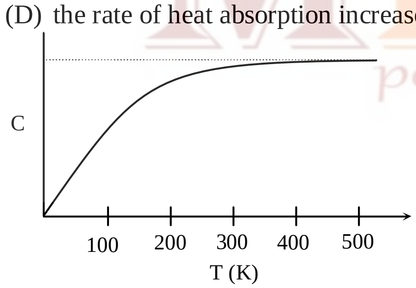
$$\vec{E} = \frac{K\left(\rho\frac{4}{3}\pi R_1^3\right)\vec{r}_2}{R_1^3} + \frac{K\left(\rho\frac{4}{3}\pi R_2^3\right)(-\vec{r}_1)}{R_2^3}$$

$$\vec{E} = K\rho\frac{4}{3}\pi(\vec{r}_2 - \vec{r}_1)$$

$$\vec{E} = \frac{\rho}{3\varepsilon_0}\vec{d}$$

***7.** The figure shows the variation of specific heat capacity (C) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to a reasonable approximation.

(A) the rate at which heat is absorbed in the range 0-100 K varies linearly with temperature T.
(B) heat absorbed in increasing the temperature from 0-100 K is less than the heat required for increasing the temperature from 400 – 500 K.
(C) there is no change in the rate of heat absorption in range 400 – 500 K.
(D) the rate of heat absorption increases in the range 200 – 300 K.



Sol. **(A, B, C, D)**

Option (A) is correct because the graph between (0 – 100 K) appears to be a straight line upto a reasonable approximation.

Option (B) is correct because area under the curve in the temperature range (0 – 100 K) is less than in range (400 – 500 K.)

Option (C) is correct because the graph of C versus T is constant in the temperature range (400 – 500 K)

Option (D) is correct because in the temperature range (200 – 300 K) specific heat capacity increases with temperature.

8. The radius of the orbit of an electron in a Hydrogen-like atom is 4.5 a_0 where a_0 is the Bohr radius. Its orbital angular momentum is $\frac{3h}{2\pi}$. It is given that h is Planck's constant and R is Rydberg constant. The possible wavelength(s), when the atom de-excites, is (are)

(A) $\frac{9}{32R}$ (B) $\frac{9}{16R}$ (C) $\frac{9}{5R}$ (D) $\frac{4}{3R}$

Sol. **(A, C)**

Given data

$$4.5a_0 = a_0\frac{n^2}{Z} \qquad \ldots(i)$$

$$\frac{nh}{2\pi} = \frac{3h}{2\pi} \qquad \ldots(ii)$$

So n = 3 and z = 2

So possible wavelength are

$$\frac{1}{\lambda_1} = RZ^2\left[\frac{1}{1^2}-\frac{1}{3^2}\right] \Rightarrow \lambda_1 = \frac{9}{32R}$$

$$\frac{1}{\lambda_2} = RZ^2\left[\frac{1}{1^2}-\frac{1}{2^2}\right] \Rightarrow \lambda_2 = \frac{1}{3R}$$

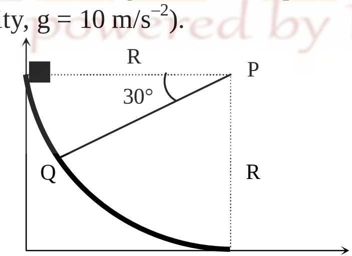
$$\frac{1}{\lambda_3} = RZ^2\left[\frac{1}{2^2}-\frac{1}{3^2}\right] \Rightarrow \lambda_3 = \frac{9}{5R}$$

SECTION – 2 : (Paragraph Type)

This section contains **4 paragraphs** each describing theory, experiment, date etc. **Eight questions** relate to four paragraphs with two questions on each paragraph. Each question of paragraph has **only one correct answer** along the four choice (A), (B), (C) and (D).

Paragraph for Questions 9 to 10

A small block of mass 1 kg is released from rest at the top of a rough track. The track is circular arc of radius 40 m. The block slides along the track without toppling and a frictional force acts on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point Q, as shown in the figure, below, is 150 J. (Take the acceleration due to gravity, g = 10 m/s^{-2}).



*9. The speed of the block when it reaches the point Q is
(A) 5 ms^{-1} (B) 10 ms^{-1} (C) $10\sqrt{3}$ ms^{-1} (D) 20 ms^{-1}

Sol. **(B)**
Using work energy theorem

$$mg R \sin 30° + W_f = \frac{1}{2}mv^2$$

$$200 - 150 = \frac{v^2}{2}$$

v = 10 m/s

*10. The magnitude of the normal reaction that acts on the block at the point Q is
(A) 7.5 N (B) 8.6 N (C) 11.5 N (D) 22.5 N

Sol. **(A)**

$$N - mg \cos 60° = \frac{mv^2}{R}$$

$$N = 5 + \frac{5}{2} = 7.5 \text{ Newton.}$$

Paragraph for Questions 11 to 12

A thermal power plant produces electric power of 600 kW at 4000 V, which is to be transported to a place 20 km away from the power plant for consumers' usage. It can be transported either directly with a cable of large current carrying capacity or by using a combination of step-up and step-down transformers at the two ends. The drawback of the direct transmission is the large energy dissipation. In the method using transformers, the dissipation is much smaller. In this method, a step-up transformer is used at the plant side so that the current is reduced to a smaller value. At the consumers' end, a step-down transformer is used to supply power to the consumers at the specified lower voltage. It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with the power factor unity. All the currents and voltage mentioned are rms values.

11. If the direct transmission method with a cable of resistance 0.4 Ω km^{-1} is used, the power dissipation (in %) during transmission is
(A) 20 (B) 30 (C) 40 (D) 50

Sol. **(B)**
For direct transmission

$$P = i^2R = (150)^2(0.4\times 20) = 1.8\times 10^5 \text{ w}$$

$$\text{fraction(in \%)} = \frac{1.8\times 10^5}{6\times 10^5}\times 100 = 30\%$$

12. In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1 : 10. If the power to the consumers has to be supplied at 200 V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is
(A) 200 : 1 (B) 150 : 1 (C) 100 : 1 (D) 50 : 1

Sol. **(A)**

$$\frac{40000}{200} = 200$$

Paragraph for Questions 13 to 14

A point Q is moving in a circular orbit of radius R in the x-y plane with an angular velocity ω. This can be considered as equivalent to a loop carrying a steady current $\frac{Q\omega}{2\pi}$. A uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The application of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around closed loop. It is known that, for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionality constant γ.

13. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change, is

(A) $\frac{BR}{4}$ (B) $\frac{BR}{2}$ (C) BR (D) 2BR

Sol. **(B)**

$E(2\pi R) = \pi R^2 \frac{dB}{dt}$

$E = \frac{RB}{2}$

14. The change in the magnetic dipole moment associated with the orbit, at the end of time interval of the magnetic field change, is

(A) $-\gamma BQR^2$ (B) $-\gamma\frac{BQR^2}{2}$ (C) $\gamma\frac{BQR^2}{2}$ (D) γBQR^2

Sol. **(B)**

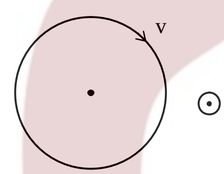
$\Delta L = \int \tau dt$

$= Q\left(\frac{R}{2}B\right)R(1)$

$= \frac{QR^2B}{2}$, in magnitude

$\Delta\mu = \gamma\Delta L$

$= -\gamma\frac{BQR^2}{2}$ (taking into account the direction)



Paragraph for Questions 15 to 16

The mass of nucleus $^{A}_{Z}X$ is less than the sum of the masses of (A-Z) number of neutrons and Z number of protons in the nucleus. The energy equivalent to the corresponding mass difference is known as the binding energy of the nucleus. A heavy nucleus of mass M can break into two light nuclei of mass m_1 and m_2 only if $(m_1 + m_2) < M$. Also two light nuclei of masses m_3 and m_4 can undergo complete fusion and form a heavy nucleus of mass M′ only if $(m_3 + m_4) > M'$. The masses of some neutral atoms are given in the table below:

$^{1}_{1}H$	1.007825 u	$^{2}_{1}H$	2.014102 u	$^{3}_{1}H$	3.016050 u	$^{4}_{2}He$	4.002603 u
$^{6}_{3}Li$	6.015123 u	$^{7}_{3}Li$	7.016004 u	$^{70}_{30}Zn$	69.925325 u	$^{82}_{34}Se$	81.916709 u
$^{152}_{64}Gd$	151.919803 u	$^{206}_{82}Pb$	205.974455 u	$^{209}_{83}Bi$	208.980388 u	$^{210}_{84}Po$	209.982876 u

15. The correct statement is

(A) The nucleus ${}^{6}_{3}\text{Li}$ can emit an alpha particle

(B) The nucleus ${}^{210}_{84}\text{Po}$ can emit a proton.

(C) Deuteron and alpha particle can undergo complete fusion.

(D) The nuclei ${}^{70}_{30}\text{Zn}$ and ${}^{82}_{34}\text{Se}$ can undergo complete fusion.

Sol. **(C)**

${}^{6}_{3}\text{Li} \rightarrow {}^{4}_{2}\text{He} + {}^{2}_{1}\text{H}$

$\frac{Q}{C^2} = 6.015123 - 4.002603 - 2.014102$

$0 = -0.001582 < 0$

So no α-decay is possible

${}^{210}_{84}\text{P}_0 \rightarrow {}^{1}_{1}\text{H} + {}^{209}_{83}\text{Bi}$

$\frac{Q}{C^2} = 209.9828766 - 1.007825 - 208.980388 = -0.005337 < 0$

So, this reaction is not possible

${}^{2}_{1}\text{H} + {}^{4}_{2}\text{He} \rightarrow {}^{6}_{3}\text{Li}$

$\frac{Q}{C^2} = 2.014102 + 4.002603 - 6.015123 = 0.001582 > 0$

So, this reaction is possible

${}^{70}_{30}\text{Zn} + {}^{82}_{34}\text{Se} \rightarrow {}^{152}_{64}\text{Gd}$

$\frac{Q}{C^2} = 69.925325 + 81.916709 - 151.919803 = -0.077769 < 0$

So this reaction is not possible

16. The kinetic energy (in keV) of the alpha particle, when the nucleus ${}^{210}_{84}\text{Po}$ at rest undergoes alpha decay, is

(A) 5319 (B) 5422 (C) 5707 (D) 5818

Sol. **(A)**

${}^{210}_{84}\text{Po} \rightarrow {}^{4}_{2}\text{He} + {}^{206}_{82}\text{Pb}$

$Q = (209.982876 - 4.002603 - 205.97455)C^2$

$= 5.422$ MeV

from conservation of momentum

$\sqrt{2K_1(4)} = \sqrt{2K_2(206)}$

$4K_1 = 206K_2$

$\therefore K_1 = \frac{103}{2}K_2$

$K_1 + K_2 = 5.422$

$K_1 + \frac{2}{103}K_1 = 5.422$

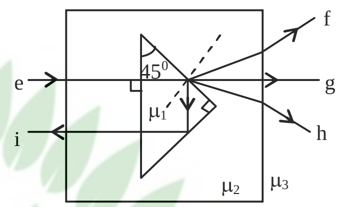
$\Rightarrow \frac{105}{103}K_1 = 5.422$

$\therefore K_1 = 5.319$ MeV = 5319 KeV

SECTION – 3 (Matching List Type)

This section contains **4 multiple choice questions.** Each question has matching lists. The codes for the lists have choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

17. A right angled prism of refractive index μ_1 is placed in a rectangular block of refractive index μ_2, which is surrounded by a medium of refractive index μ_3, as shown in the figure. A ray of light 'e' enters the rectangular block at normal incidence. Depending upon the relationships between μ_1, μ_2 and μ_3, it takes one of the four possible paths 'ef', 'eg', 'eh', or 'ei'.



Match the paths in List I with conditions of refractive indices in List II and select the correct answer using the codes given below the lists:

	List I		List II
P.	$e \to f$	1.	$\mu_1 > \sqrt{2}\,\mu_2$
Q.	$e \to g$	2.	$\mu_2 > \mu_1$ and $\mu_2 > \mu_3$
R.	$e \to h$	3.	$\mu_1 = \mu_2$
S.	$e \to i$	4.	$\mu_2 < \mu_1 < \sqrt{2}\,\mu_2$ and $\mu_2 > \mu_3$

Codes:

	P	Q	R	S
(A)	2	3	1	4
(B)	1	2	4	3
(C)	4	1	2	3
(D)	2	3	4	1

Sol. **(D)**

P. → (2) ; Q. → (3); R. → (4); S. → (1)

P. $\mu_2 > \mu_1 \ldots$ (towards normal)

$\mu_2 > \mu_3 \ldots$ (away from normal)

Q. $\mu_1 = \mu_2 \ldots$ (No change in path)

$\angle i = 0 \Rightarrow \angle r = 0$ on the block.

R. $\mu_1 > \mu_2 \ldots$ (Away from the normal)

$\mu_2 > \mu_3 \ldots$ (Away from the normal)

$\mu_1 \times \frac{1}{\sqrt{2}} = \mu_2 \sin r \Rightarrow \sin r = \frac{\mu_1}{\sqrt{2}\mu_2}$. Since $\sin r < 1 \Rightarrow \mu_1 < \sqrt{2}\mu_2$

S. For TIR : $45^0 > C \Rightarrow \sin 45^0 > \sin C \Rightarrow \frac{1}{\sqrt{2}} > \frac{\mu_2}{\mu_1} \Rightarrow \mu_1 > \sqrt{2}\mu_2$

*18. Match List I with List II and select the correct answer using the codes given below the lists:

	List I		List II
P.	Boltzmann Constant	1.	$[ML^2T^{-1}]$
Q.	Coefficient of viscosity	2.	$[ML^{-1}T^{-1}]$
R.	Plank Constant	3.	$[MLT^{-3}K^{-1}]$
S.	Thermal conductivity	4.	$[ML^2T^{-2}K^{-1}]$

Codes:

	P	Q	R	S
(A)	3	1	2	4
(B)	3	2	1	4
(C)	4	2	1	3
(D)	4	1	2	3

Sol. **(C)**

P. $\rightarrow$ (4) ; Q. $\rightarrow$ (2); R. $\rightarrow$ (1); S. $\rightarrow$ (3)

P. $KE = \frac{3}{2}K'T \Rightarrow [ML^2T^{-2}] = K'[K] \Rightarrow K' = [ML^2T^{-2}K^{-1}]$

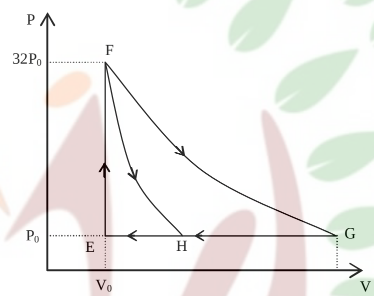
Q. $F = 6\pi\eta rv \Rightarrow [MLT^{-2}] = \eta[L][LT^{-1}] \Rightarrow \eta = [ML^{-1}T^{-1}]$

R. $E = hf \Rightarrow [ML^2T^{-2}] = \frac{h}{[T]} \Rightarrow h = [ML^2T^{-1}]$

S. $\frac{dQ}{dt} = \frac{K'A(\Delta T)}{\Delta x} \Rightarrow \frac{[ML^2T^{-2}]}{[T]} = \frac{k[L^2][K']}{[L]}$

$K' = [MLT^{-3}K^{-1}]$

*19. One mole of mono-atomic ideal gas is taken along two cyclic processes E→F→G→E and E→F→H→E as shown in the PV diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic.



Match the paths in List I with the magnitudes of the work done in List II and select the correct answer using the codes given below the lists.

	List I		List II
P.	$G \rightarrow E$	1.	$160\ P_0V_0\ \ln 2$
Q.	$G \rightarrow H$	2.	$36\ P_0V_0$
R.	$F \rightarrow H$	3.	$24\ P_0V_0$
S.	$F \rightarrow G$	4.	$31\ P_0\ V_0$

Codes:

	P	Q	R	S
(A)	4	3	2	1
(B)	4	3	1	2
(C)	3	1	2	4
(D)	1	3	2	4

Sol. **(A)**

P. $\rightarrow$ (4) ; Q. $\rightarrow$ (3); R. $\rightarrow$ (2); S. $\rightarrow$ (1)

Apply $PV^{1+2/3}$ = constant for F to H.

$(32P_0)\,V_0^{5/3} = P_0\,V_H^{5/3} \Rightarrow V_H = 8V_0$

For path FG PV = constant

$\Rightarrow (32P_0)V_0 = P_0V_G \Rightarrow V_G = 32V_0$

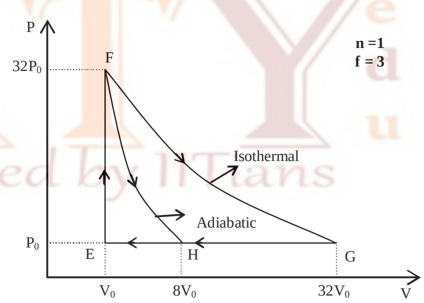
Work done in GE = $31\ P_0V_0$

Work done in GH = $24\ P_0V_0$

Work done in FH = $\frac{P_HV_H - P_FV_F}{(-2/f)} = 36P_0V_0$

Work done in FG = $RT \ln\left(\frac{V_G}{V_F}\right)$

$= 160P_0V_0\ln 2.$



20. Match List I of the nuclear processes with List II containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists:

	List I		List II
P.	Alpha decay	1.	${}^{15}_{8}O \to {}^{15}_{7}N + ...$
Q.	β+ decay	2.	${}^{238}_{92}U \to {}^{234}_{90}Th + ...$
R.	Fission	3.	${}^{185}_{83}Bi \to {}^{184}_{82}Pb + ...$
S.	Proton emission	4.	${}^{239}_{94}Pu \to {}^{140}_{57}La + ...$

Codes:

	P	Q	R	S
(A)	4	2	1	3
(B)	1	3	2	4
(C)	2	1	4	3
(D)	4	3	2	1

Sol. **(C)**

P. → (2) ; Q. → (1); R. → (4); S. → (3)

${}^{15}_{8}O \to {}^{15}_{7}N + {}^{0}_{1}\beta$ (Beta decay)

${}^{238}_{92}U \to {}^{234}_{90}Th + {}^{4}_{2}He$ (Alpha decay)

${}^{185}_{83}Bi \to {}^{184}_{82}Pb + {}^{1}_{1}H$ (Proton emission)

${}^{239}_{94}Ph \to {}^{140}_{57}La + {}^{99}_{37}Rb$ (fission)

PART - II: CHEMISTRY

SECTION –1 (One or more options correct Type)

This section contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** are correct.

*21. The K_{sp} of Ag_2CrO_4 is 1.1×10^{-12} at 298K. The solubility (in mol/L) of Ag_2CrO_4 in a 0.1M $AgNO_3$ solution is

(A) 1.1×10^{-11}
(B) 1.1×10^{-10}
(C) 1.1×10^{-12}
(D) 1.1×10^{-9}

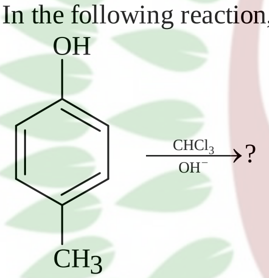
Sol. **(B)**

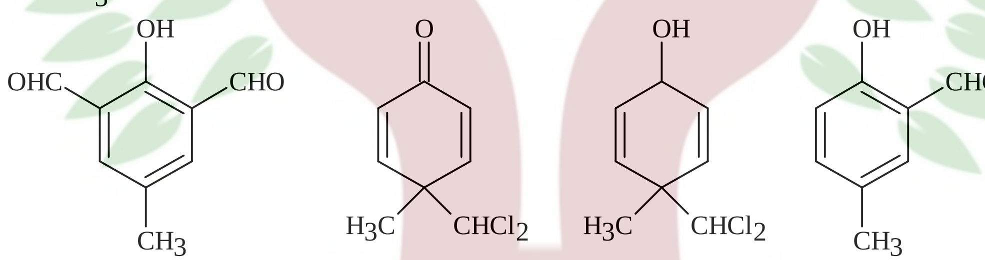
$$K_{sp} = 1.1 \times 10^{-12} = \left[Ag^+\right]^2 \left[CrO_4^{-2}\right]$$

$$1.1 \times 10^{-12} = [0.1]^2 [s]$$

$$s = 1.1 \times 10^{-10}$$

22. In the following reaction, the product(s) formed is(are)

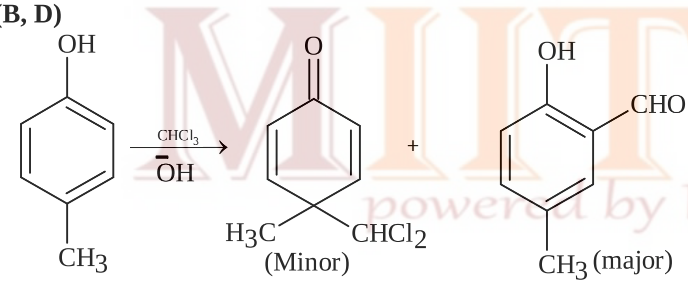




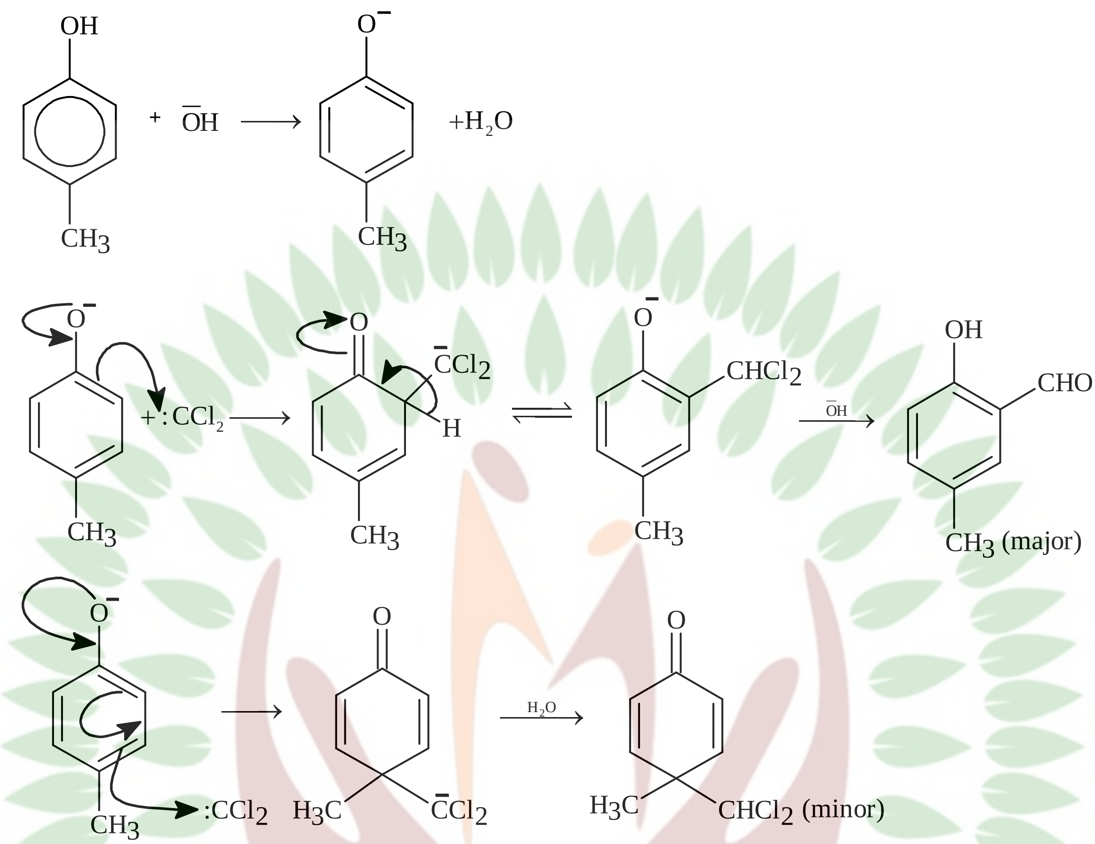
P Q R S

(A) P(major)
(B) Q(minor)
(C) R(minor)
(D) S(major)

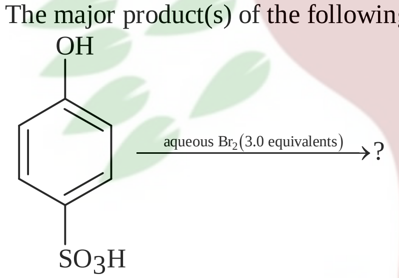
Sol. **(B, D)**

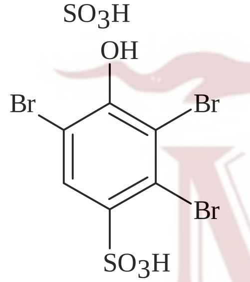


$$CHCl_3 + \overline{O}H \longrightarrow :CCl_2 + H_2O + Cl^-$$

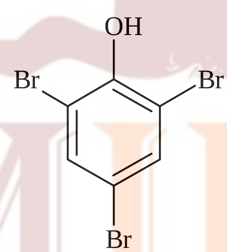


23. The major product(s) of the following reaction is (are)

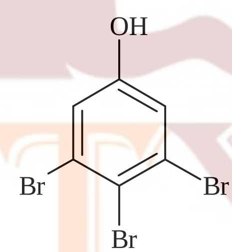




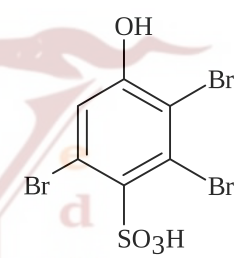
P



Q



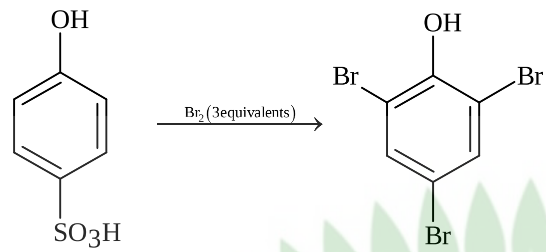
R



S

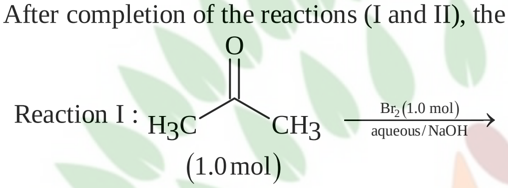
(A) P
(B) Q
(C) R
(D) S

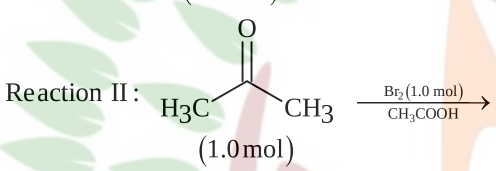
Sol. **(B)**

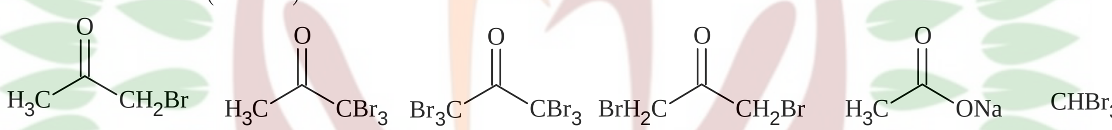


(Q)

24. After completion of the reactions (I and II), the organic compound(s) in the reaction mixtures is(are)



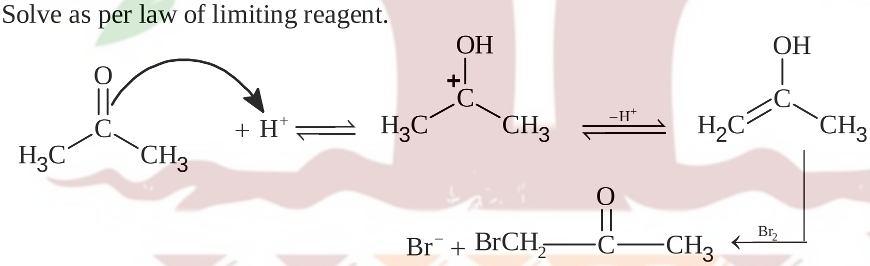




P **Q** **R** **S** **T** **U**

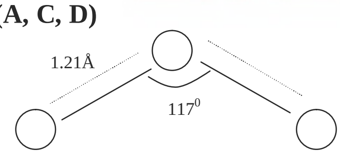
(A) Reaction I: P and Reaction II : P
(B) Reaction I: U, acetone and Reaction II:Q, acetone
(C) Reaction I : T, U, acetone and Reaction II : P
(D) Reaction I : R, acetone and Reaction II : S, acetone

Sol. **(C)**
Solve as per law of limiting reagent.



25. The correct statement(s) about O_3 is(are)
(A) O–O bond lengths are equal.
(B) Thermal decomposition of O_3 is endothermic.
(C) O_3 is diamagnetic in nature.
(D) O_3 has a bent structure.

Sol. **(A, C, D)**



*26. In the nuclear transmutation

$${}_4^9Be + X \longrightarrow {}_4^8Be + Y$$

(X, Y) is (are)

(A) (γ, n)
(B) (p, D)
(C) (n, D)
(D) (γ, p)

Sol. **(A, B)**

$${}_4^9Be + \gamma \longrightarrow {}_4^8Be + {}_0^1n$$

$${}_4^9Be + {}_1^1P \longrightarrow {}_4^8Be + {}_1^2H$$

Hence (A) and (B) are correct

27. The carbon–based reduction method is NOT used for the extraction of

(A) tin from SnO_2
(B) iron from Fe_2O_3
(C) aluminium from Al_2O_3
(D) magnesium from $MgCO_3.CaCO_3$

Sol. **(C, D)**

Fe_2O_3 and SnO_2 undergoes C reduction. Hence (C) and (D) are correct.

*28. The thermal dissociation equilibrium of $CaCO_3$(s) is studied under different conditions.

$$CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$$

For this equilibrium, the correct statement(s) is(are)

(A) ΔH is dependent on T
(B) K is independent of the initial amount of $CaCO_3$
(C) K is dependent on the pressure of CO_2 at a given T
(D) ΔH is independent of the catalyst, if any

Sol. **(A, B, D)**

For the equilibrium $CaCO_3(s) \rightleftharpoons CaO(S) + CO_2(g)$. The equilibrium constant (K) is independent of initial amount of $CaCO_3$ where as at a given temperature is independent of pressure of CO_2. ΔH is independent of catalyst and it depends on temperature.

Hence (A), (B) and (D) are correct.

SECTION-2 (Paragraph Type)

This section contains **4 paragraphs** each describing theory, experiment, data etc. **Eight questions** relate to four paragraphs with two questions on each paragraph. Each question of a paragraph has **only one correct answer** among the four choices (A), (B), (C) and (D).

Paragraph for Question Nos. 29 and 30

An aqueous solution of a mixture of two inorganic salts, when treated with dilute HCl, gave a precipitate **(P)** and a filtrate **(Q).** The precipitate **P** was found to dissolve in hot water. The filtrate **(Q)** remained unchanged, when treated with H_2S in a dilute mineral acid medium. However, it gave a precipitate **(R)** with H_2S in an ammoniacal medium. The precipitate **R** gave a coloured solution **(S),** when treated with H_2O_2 in an aqueous NaOH medium.

29. The precipitate **P** contains

(A) Pb^{2+}
(B) Hg_2^{2+}
(C) Ag^{+}
(D) Hg^{2+}

Sol. **(A)**

30. The coloured solution **S** contains
(A) $Fe_2(SO_4)_3$ (B) $CuSO_4$
(C) $ZnSO_4$ (D) Na_2CrO_4

Sol. **(D)**

Solution for the Q. No. 29 to 30.

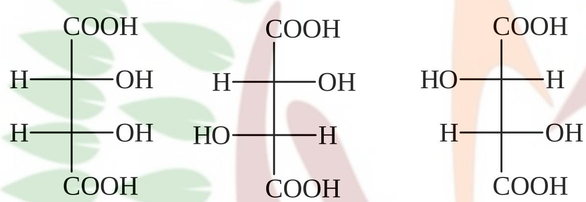
$$Pb^{+2} + 2HCl \longrightarrow PbCl_2 \xrightarrow[\text{Water}]{\text{Hot}} \text{soluble.}$$

$$Cr^{+3} \xrightarrow[\text{ammoniacal}]{H_2S} Cr(OH)_3 \downarrow$$

$$Cr(OH)_3 \xrightarrow[H_2O_2]{NaOH} \underset{\text{Yellow solution}}{Na_2CrO_4}$$

Paragraph for Question Nos. 31 to 32

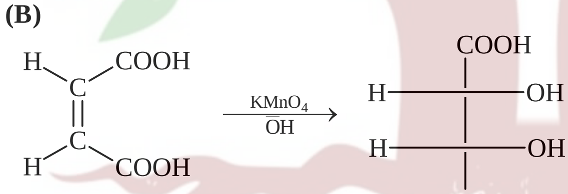
P and **Q** are isomers of dicarboxylic acid $C_4H_4O_4$. Both decolorize Br_2/H_2O. On heating, **P** forms the cyclic anhydride.
Upon treatment with dilute alkaline $KMnO_4$, **P** as well as **Q** could produce one or more than one from **S, T** and **U.**



S **T** **U**

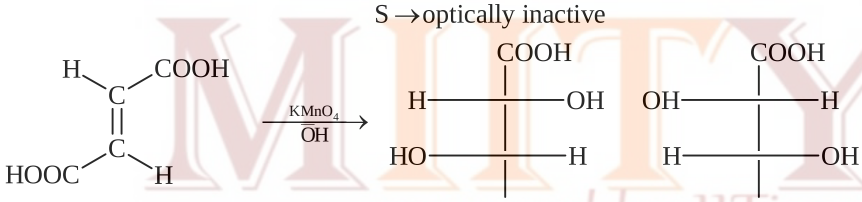
*31. Compounds formed from **P** and **Q** are, respectively
(A) Optically active **S** and optically active pair **(T, U)**
(B) Optically inactive **S** and optically inactive pair **(T, U)**
(C) Optically active pair **(T, U)** and optically active **S**
(D) Optically inactive pair **(T, U)** and optically inactive **S**

Sol. **(B)**



P

S →optically inactive



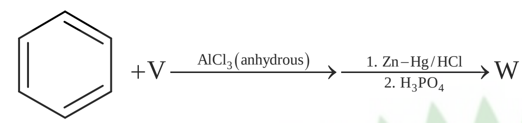
Q

T U

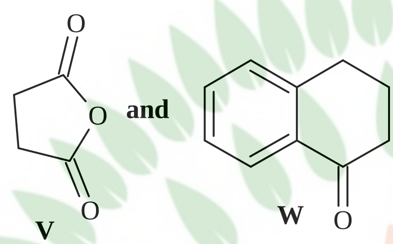
optically inactive pair

*32. In the following reaction sequences **V** and **W** are, respectively

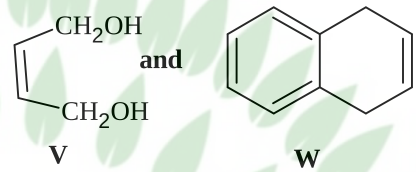
$Q \xrightarrow[\Delta]{H_2/Ni} V$



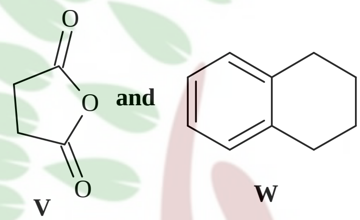
(A)



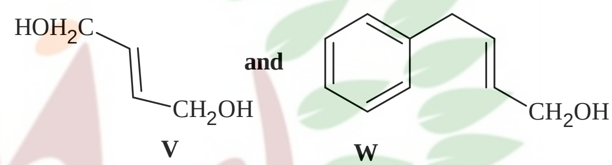
(B)



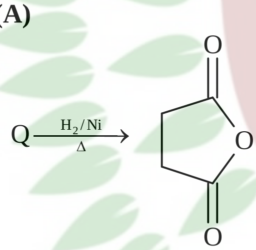
(C)

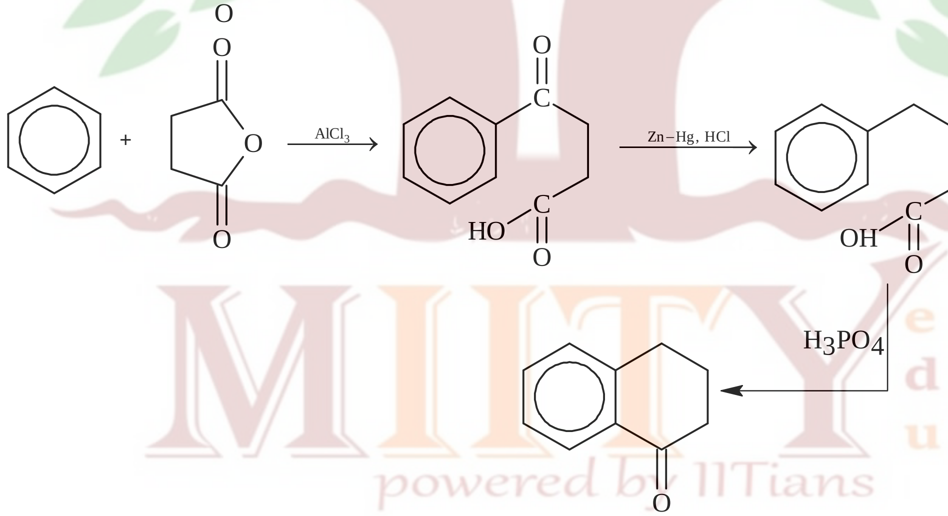


(D)



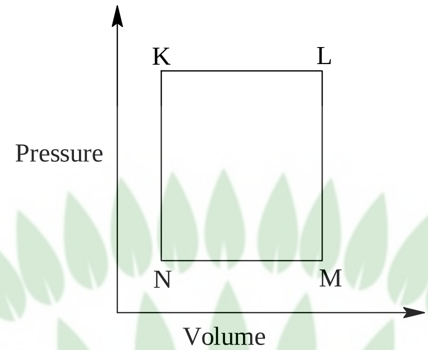
Sol. **(A)**





Paragraph for Question Nos. 33 to 34

A fixed mass 'm' of a gas is subjected to transformation of states from K to L to M to N and back to K as shown in the figure



*33. The succeeding operations that enable this transformation of states are

(A) Heating, cooling, heating, cooling
(B) Cooling, heating, cooling, heating
(C) Heating, cooling, cooling, heating
(D) Cooling, heating, heating, cooling

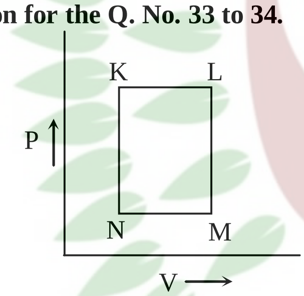
Sol. **(C)**

*34. The pair of isochoric processes among the transformation of states is

(A) K to L and L to M
(B) L to M and N to K
(C) L to M and M to N
(D) M to N and N to K

Sol. **(B)**

Solution for the Q. No. 33 to 34.



K – L heating, isobaric
L – M cooling, isochoric
M – N cooling, isobaric
N – K heating, isochoric

Paragraph for Question Nos. 35 to 36

The reactions of Cl_2 gas with cold-dilute and hot-concentrated NaOH in water give sodium salts of two (different) oxoacids of chlorine, **P** and **Q,** respectively. The Cl_2 gas reacts with SO_2 gas, in presence of charcoal, to give a product **R.** **R** reacts with white phosphorus to give a compound **S.** On hydrolysis, **S** gives an oxoacid of phosphorus, **T**.

35. **P** and **Q,** respectively, are the sodium salts of

(A) hypochlorus and chloric acids
(B) hypochlorus and chlorus acids
(C) chloric and perchloric acids
(D) chloric and hypochlorus acids

Sol. **(A)**

36. **R, S** and **T,** respectively, are

(A) SO_2Cl_2, PCl_5 and H_3PO_4
(B) SO_2Cl_2, PCl_3 and H_3PO_3
(C) $SOCl_2$, PCl_3 and H_3PO_2
(D) $SOCl_2$, PCl_5 and H_3PO_4

Sol. **(A)**

Solution for the Q. No. 35 to 36

$$\underset{(\text{Cold}+\text{dil})}{2NaOH}+Cl_2 \longrightarrow NaCl+\underset{P}{NaClO}+H_2O$$

$$\underset{(\text{Hot}+\text{conc.})}{6NaOH}+3Cl_2 \longrightarrow 5NaCl+\underset{Q}{NaClO_3}+3H_2O$$

$$SO_2+Cl_2 \xrightarrow{\text{Charcoal}} \underset{R}{SO_2Cl_2}$$

$$SO_2Cl_2+P_4 \longrightarrow \underset{S}{PCl_5}+SO_2$$

$$PCl_5+H_2O \longrightarrow \underset{T}{H_3PO_4}+HCl$$

SECTION – 3: (Matching List Type)

This section contains **4 multiple choice questions. Each question has matching lists.** The codes for the lists have choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

---

37. The unbalanced chemical reactions given in List – I show missing reagent or condition (?) which are provided in List – II. Match List – I with List – II and select the correct answer using the code given below the lists:

	List – I		List - II
(P)	$PbO_2+H_2SO_4 \xrightarrow{?} PbSO_4+O_2+\text{other product}$	(1)	NO
(Q)	$Na_2S_2O_3+H_2O \xrightarrow{?} NaHSO_4+\text{other product}$	(2)	I_2
(R)	$N_2H_4 \xrightarrow{?} N_2+\text{other product}$	(3)	Warm
(S)	$XeF_2 \xrightarrow{?} Xe+\text{other product}$	(4)	Cl_2

Codes:

	P	Q	R	S
(A)	4	2	3	1
(B)	3	2	1	4
(C)	1	4	2	3
(D)	3	4	2	1

Sol. **(D)**

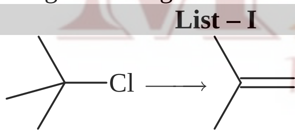
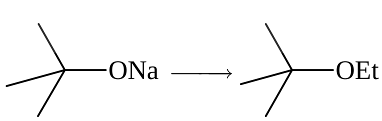
(P) $PbO_2+H_2SO_4 \xrightarrow{\Delta} PbSO_4+H_2O+\frac{1}{2}O_2$

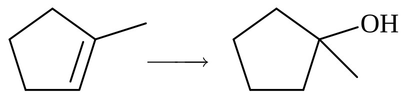
(Q) $2Na_2S_2O_3+Cl_2+2H_2O \longrightarrow 2NaCl+2NaHSO_4+2S$

(R) $N_2H_4+2I_2 \longrightarrow N_2+4HI$

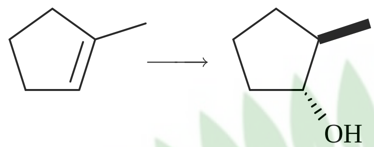
(S) $XeF_2+2NO \longrightarrow Xe+2NOF$

*38. Match the chemical conversions in List – I with appropriate reagents in List – II and select the correct answer using the code given below the lists:

	List – I		List - II
(P)		(1)	(i) $Hg(OAc)_2$; (ii) $NaBH_4$
(Q)		(2)	NaOEt

(R) 

(3) Et-Br

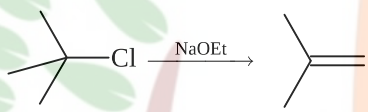
(S) 

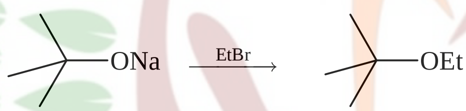
(4) (i) BH_3; (ii) H_2O_2/NaOH

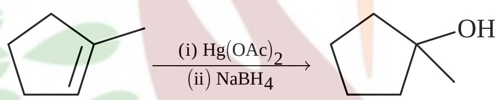
Codes:

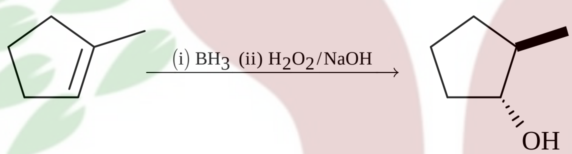
	P	Q	R	S
(A)	2	3	1	4
(B)	3	2	1	4
(C)	2	3	4	1
(D)	3	2	4	1

Sol. **(A)**

(P) 

(Q) 

(R) 

(S) 

39. An aqueous solution of X is added slowly to an aqueous solution of Y as shown in List – I. The variation in conductivity of these reactions in List – II. Match List – I with List – II and select the correct answer using the code given below the lists:

	List – I		List - II
(P)	$\underset{X}{(C_2H_5)_3N} + \underset{Y}{CH_3COOH}$	(1)	Conductivity decreases and then increases
(Q)	$\underset{X}{KI(0.1M)} + \underset{Y}{AgNO_3(0.01M)}$	(2)	Conductivity decreases and then does not change much
(R)	$\underset{X}{CH_3COOH} + \underset{Y}{KOH}$	(3)	Conductivity increases and then does not change much
(S)	$\underset{X}{NaOH} + \underset{Y}{HI}$	(4)	Conductivity does not change much and then increases

Codes:

	P	Q	R	S
(A)	3	4	2	1
(B)	4	3	2	1
(C)	2	3	4	1
(D)	1	4	3	2

Sol. **(A)**

(P) $\underset{X}{(C_2H_5)_3N} + \underset{Y}{CH_3COOH} \longrightarrow (C_2H_5)_3NH^+CH_3COO^-$

Initially conductivity increases due to ion formation after that it becomes practically constant because X alone can not form ions. Hence (3) is the correct match.

(Q) $\underset{X}{KI(0.1\,M)} + \underset{Y}{AgNO_3(0.01M)} \longrightarrow AgI\downarrow + KNO_3$

Number of ions in the solution remains constant until all the $AgNO_3$ precipitated as AgI. Thereafter conductance increases due to increases in number of ions. Hence (4) is the correct match.

(R) Initially conductance decreases due to the decrease in the number of $\overline{O}H$ ions thereafter it slowly increases due to the increases in number of H^+ ions. Hence (2) is the correct match.

(S) Initially it decreases due to decrease in H^+ ions and then increases due to the increases in $\overline{O}H$ ions. Hence (1) is the correct match.

40. The standard reduction potential data at 25°C is given below:

$E^\circ(Fe^{3+}, Fe^{2+}) = +0.77V;$

$E^\circ(Fe^{2+}, Fe) = -0.44V$

$E^\circ(Cu^{2+}, Cu) = +0.34V;$

$E^\circ(Cu^{+}, Cu) = +0.52V$

$E^\circ[O_2(g) + 4H^+ + 4e^- \rightarrow 2H_2O] = +1.23V;$

$E^\circ[O_2(g) + 2H_2O + 4e^- \rightarrow 4OH^-] = +0.40V$

$E^\circ(Cr^{3+}, Cr) = -0.74V;$

$E^\circ(Cr^{2+}, Cr) = -0.91V$

Match E^0 of the redox pair in List – I with the values given in List – II and select the correct answer using the code given below the lists:

(P)	$E^\circ(Fe^{3+}, Fe)$	(1)	–0.18 V
(Q)	$E^\circ(4H_2O \rightleftharpoons 4H^+ + 4OH^-)$	(2)	–0.4 V
(R)	$E^\circ(Cu^{2+} + Cu \longrightarrow 2Cu^+)$	(3)	–0.04 V
(S)	$E^\circ(Cr^{3+}, Cr^{2+})$	(4)	–0.83 V

Codes:

	P	Q	R	S
(A)	4	1	2	3
(B)	2	3	4	1
(C)	1	2	3	4
(D)	3	4	1	2

Sol. **(D)**

(P) $\Delta G^o_{Fe^{3+}/Fe} = \Delta G^o_{Fe^{3+}/Fe^{2+}} + \Delta G^o_{Fe^{2+}/Fe}$

$\Rightarrow -3 \times FE^o_{(Fe^{+3}/Fe)} = -1 \times FE^o_{(Fe^{+3}/Fe^{+2})} + (-2 \times FE^o_{Fe^{+2}/Fe})$

$\Rightarrow E^o_{Fe^{+3}/Fe} = -0.04\ V$

(Q) $O_2(g) + 2H_2O + 4e^- \longrightarrow 4\overset{-}{O}H \quad E^o = 0.40\ V$ ... (i)

$2H_2O \longrightarrow O_2(g) + 4H^+ + 4e^- \quad E^o = -1.23\ V$ .. (ii)

So $4H_2O \rightleftharpoons 4H^+ + 4\overset{-}{O}H$ ... (iii)

E^o for IIIrd reduction = 0.40 – 1.23 = - 0.83 V.

(R) $\Delta G^o_{(Cu^{+2}/Cu)} = \Delta G^o_{(Cu^{+2}/Cu^+)} + \Delta G^o_{(Cu^+/Cu)}$

$-2 \times FE^o_{Cu^{+2}/Cu} = -1 \times FE^o_{Cu^{+2}/Cu^+} + \left(-1 \times F \times E^o_{Cu^+/Cu}\right)$

$\Rightarrow E^o_{Cu^{+2}/Cu} = -0.18\ V$.

(S) $\Delta G^o_{Cr^{+3}/Cr^{+2}} = \Delta G^o_{Cr^{+3}/Cr} + \Delta G^o_{Cr/Cr^{+2}}$

$-1 \times F \times E^o_{Cr^{+3}/Cr^{+2}} = -3 \times F \times E^o_{Cr^{+3}/Cr} + \left(-2 \times F \times E^o_{Cr/Cr^{+2}}\right)$

$\Rightarrow E^o_{Cr^{+3}/Cr^{+2}} = -0.4\,V$.

PART - III: MATHEMATICS

SECTION – 1 : (One or more option correct Type)

This section contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

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41. For a ∈ R (the set of all real numbers), a ≠ −1, $\lim_{n\to\infty}\frac{\left(1^a+2^a+\ldots+n^a\right)}{(n+1)^{a-1}\left[(na+1)+(na+2)+\ldots+(na+n)\right]}=\frac{1}{60}$.

Then a =

(A) 5
(B) 7
(C) $\frac{-15}{2}$
(D) $\frac{-17}{2}$

Sol. **(B, D)**

Required limit = $\frac{\int_0^1 x^a\,dx}{\int_0^1 (a+x)\,dx}=\frac{2}{(2a+1)(a+1)}=\frac{2}{120}$

$\Rightarrow$ a = 7 or $-\frac{17}{2}$.

*42. Circle(s) touching x-axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ on y-axis is (are)

(A) $x^2 + y^2 - 6x + 8y + 9 = 0$
(B) $x^2 + y^2 - 6x + 7y + 9 = 0$
(C) $x^2 + y^2 - 6x - 8y + 9 = 0$
(D) $x^2 + y^2 - 6x - 7y + 9 = 0$

Sol. **(A), (C)**

Equation of circle can be written as

$(x-3)^2 + y^2 + \lambda(y) = 0$

$\Rightarrow x^2 + y^2 - 6x + \lambda y + 9 = 0.$

Now, $(\text{radius})^2 = 7 + 9 = 16$

$\Rightarrow 9+\frac{\lambda^2}{4}-9=16$

$\Rightarrow \lambda^2 = 64 \Rightarrow \lambda = \pm 8.$

$\therefore$ Equation is $x^2 + y^2 - 6x \pm 8y + 9 = 0.$

43. Two lines $L_1 : x = 5, \frac{y}{3-\alpha}=\frac{z}{-2}$ and $L_2 : x = \alpha, \frac{y}{-1}=\frac{z}{2-\alpha}$ are coplanar. Then α can take value(s)

(A) 1
(B) 2
(C) 3
(D) 4

Sol. **(A, D)**

$\frac{x-5}{0}=\frac{y-0}{3-\alpha}=\frac{z-0}{-2}$

$\frac{x-\alpha}{0}=\frac{y}{-1}=\frac{z}{2-\alpha}$

will be coplanar if shortest distance is zero

$\Rightarrow \begin{vmatrix} 5-\alpha & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$

$(5 - \alpha)(\alpha^2 - 5\alpha + 4) = 0$, $\alpha = 1, 4, 5$

so $\alpha = 1, 4$

Alternate Solution:

As $x = 5$ and $x = \alpha$ are parallel planes so the remaining two planes must be coplanar.

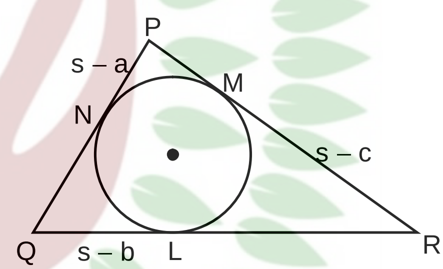
So, $\frac{3-\alpha}{-1}=\frac{-2}{2-\alpha} \Rightarrow \alpha^2 - 5\alpha + 4 = 0 \Rightarrow \alpha = 1, 4.$

*44. In a triangle PQR, P is the largest angle and $\cos P = \frac{1}{3}$. Further the incircle of the triangle touches the sides PQ, QR and RP at N, L and M respectively, such that the lengths of PN, QL and RM are consecutive even integers. Then possible length(s) of the side(s) of the triangle is (are)

(A) 16 (B) 18

(C) 24 (D) 22

Sol. **(B), (D)**



Let

$s - a = 2k - 2$, $s - b = 2k$, $s - c = 2k + 2$, $k \in I, k > 1$

Adding we get,

$s = 6k$

So, $a = 4k + 2$, $b = 4k$, $c = 4k - 2$

Now, $\cos P = \frac{1}{3}$

$\Rightarrow \frac{b^2+c^2-a^2}{2bc}=\frac{1}{3} \Rightarrow 3\,[(4k)^2 + (4k - 2)^2 - (4k + 2)^2] = 2 \times 4k\,(4k - 2)$

$\Rightarrow 3\,[16k^2 - 4\,(4k) \times 2] = 8k\,(4k - 2)$

$\Rightarrow 48k^2 - 96k = 32k^2 - 16k$

$\Rightarrow 16k^2 = 80k \Rightarrow k = 5$

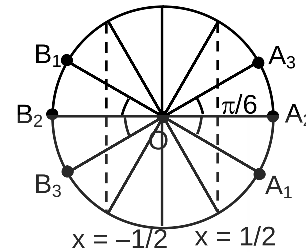
So, sides are 22, 20, 18

*45. Let $w = \frac{\sqrt{3}+i}{2}$ and $P = \{w^n : n = 1, 2, 3, \ldots\}$. Further $H_1 = \left\{z \in C : \text{Re}\, z > \frac{1}{2}\right\}$ and $H_2 = \left\{z \in C : \text{Re}\, z < \frac{-1}{2}\right\}$, where C is the set of all complex numbers. If $z_1 \in P \cap H_1$, $z_2 \in P \cap H_2$ and O represents the origin, then $\angle z_1 O z_2 =$

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$

(C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

Sol. **(C), (D)**



$w = \dfrac{\sqrt{3}+i}{2} = e^{\frac{i\pi}{6}}$, so $w^n = e^{i\left(\frac{n\pi}{6}\right)}$

Now, for z_1, $\cos\dfrac{n\pi}{6} > \dfrac{1}{2}$ and for z_2, $\cos\dfrac{n\pi}{6} < -\dfrac{1}{2}$

Possible position of z_1 are A_1, A_2, A_3 whereas of z_2 are B_1, B_2, B_3 (as shown in the figure)

So, possible value of $\angle z_1Oz_2$ according to the given options is $\dfrac{2\pi}{3}$ or $\dfrac{5\pi}{6}$.

*46. If $3^x = 4^{x-1}$, then x =

(A) $\dfrac{2\log_3 2}{2\log_3 2-1}$ (B) $\dfrac{2}{2-\log_2 3}$

(C) $\dfrac{1}{1-\log_4 3}$ (D) $\dfrac{2\log_2 3}{2\log_2 3-1}$

Sol. **(A, B, C)**

$\log_2 3^x = (x-1)\log_2 4 = 2(x-1)$

$\Rightarrow x\log_2 3 = 2x - 2$

$\Rightarrow x = \dfrac{2}{2-\log_2 3}$

Rearranging, we get

$$x = \frac{2}{2-\dfrac{1}{\log_3 2}} = \frac{2\log_3 2}{2\log_3 2-1}$$

Rearranging again,

$$x = \frac{\log_3 4}{\log_3 4-1} = \frac{\dfrac{1}{\log_4 3}}{\dfrac{1}{\log_4 3}-1} = \frac{1}{1-\log_4 3}.$$

47. Let ω be a complex cube root of unity with ω ≠ 1 and $P = [p_{ij}]$ be a n × n matrix with $p_{ij} = \omega^{i+j}$. Then $P^2 \neq 0$, when n =

(A) 57 (B) 55

(C) 58 (D) 56

Sol. **(B, C, D)**

$$P = \begin{bmatrix} \omega^2 & \omega^3 & \omega^4 & ..... & \omega^{n+2} \\ \omega^3 & \omega^4 & \omega^5 & ..... & \omega^{n+3} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \omega^{n+2} & \omega^{n+3} & ..... & ..... & \omega^{2n+4} \end{bmatrix}$$

$$P^2 = \begin{bmatrix} \omega^4+\omega^6..... & \omega^5+\omega^7+\omega^9 & ..... & ..... \\ \omega^5+\omega^7+\omega^9..... & ..... & ..... & ..... \\ \vdots & \vdots & \vdots & ..... \\ \omega^{n+4}+\omega^{n+6}..... & ..... & ..... & \omega^{2n+4}+\omega^{2n+6}..... \end{bmatrix}$$

P^2 = Null matrix if n is a multiple of 3

48. The function f(x) = 2 |x| + |x + 2| – ||x + 2| – 2 |x|| has a local minimum or a local maximum at x =

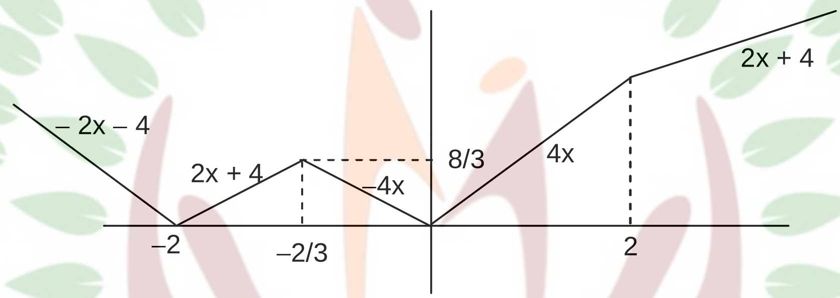
(A) –2 (B) $\frac{-2}{3}$

(C) 2 (D) $\frac{2}{3}$

Sol. **(A), (B)**

As, $\frac{f(x)+g(x)-|f(x)-g(x)|}{2}$ = Min (f (x), g (x))

$\Rightarrow \frac{2|x|+|x+2|-||x+2|-2|x||}{2}$ = Min (|2x|, |x + 2|)



According to the figure shown, points of local minima/maxima are x = –2, $\frac{-2}{3}$, 0 .

SECTION – 2 : (Paragraph Type)

This section contains 6 multiple choice questions relating to three paragraphs with two questions on each paragraph. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Questions 49 and 50

Let f : [0, 1] → R (the set of all real numbers) be a function. Suppose the function f is twice differentiable, f(0) = f(1) = 0 and satisfies $f''(x) - 2f'(x) + f(x) \geq e^x$, $x \in [0, 1]$.

49. Which of the following is true for 0 < x < 1 ?

(A) $0 < f(x) < \infty$ (B) $-\frac{1}{2} < f(x) < \frac{1}{2}$

(C) $-\frac{1}{4} < f(x) < 1$ (D) $-\infty < f(x) < 0$

Sol. **(D)**

Let $g(x) = e^{-x} f(x)$

and $g''(x) > 1 > 0$

So, g (x) is concave upward and g (0) = g (1) = 0

Hence, $g(x) < 0 \ \forall \ x \in (0, 1)$

$\Rightarrow e^{-x} f(x) < 0$

$f(x) < 0 \ \forall \ x \in (0, 1)$

Alternate Solution

$f''(x) - 2f'(x) + f(x) \geq e^x$

$\Rightarrow \left(f(x)e^{-x} - \frac{x^2}{2}\right)'' \geq 0$

Let g (x) = f (x) $e^{-x} - \frac{x^2}{2}$

g (0) = 0, g (1) = $-\frac{1}{2}$

Since g is concave up so it will always lie below the chord joining the extremities which is y = $-\frac{x}{2}$

$\Rightarrow f(x)e^{-x} - \frac{x^2}{2} < -\frac{x}{2}$

$\Rightarrow$ f (x) < $\frac{(x^2 - x)e^x}{2} < 0 \ \forall x \in (0, 1)$

50. If the function e^{-x} f(x) assumes its minimum in the interval [0, 1] at x = $\frac{1}{4}$, which of the following is true ?

(A) $f'(x) < f(x), \frac{1}{4} < x < \frac{3}{4}$

(B) $f'(x) > f(x), 0 < x < \frac{1}{4}$

(C) $f'(x) < f(x), 0 < x < \frac{1}{4}$

(D) $f'(x) < f(x), \frac{3}{4} < x < 1$

Sol. **C**

Let, g (x) = e^{-x} f (x)

As g″ (x) > 0 so g′ (x) is increasing.

So, for x < 1/4, g′ (x) < g′ (1/4) = 0

$\Rightarrow$ (f′ (x) – f (x))e^{-x} < 0

$\Rightarrow$ f′ (x) < f (x) in (0, 1/4).

Paragraph for Questions 51 and 52

Let PQ be a focal chord of the parabola y^2 = 4ax. The tangents to the parabola at P and Q meet at a point lying on the line y = 2x + a, a > 0.

*51. Length of chord PQ is

(A) 7a

(B) 5a

(C) 2a

(D) 3a

Sol. **(B)**

Let $P(at^2, 2at)$, $Q\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$ as PQ is focal chord

Point of intersection of tangents at P and Q

$\left(-a, a\left(t - \frac{1}{t}\right)\right)$

as point of intersection lies on y = 2x + a

$\Rightarrow a\left(t - \frac{1}{t}\right) = -2a + a$

$t - \frac{1}{t} = -1 \Rightarrow \left(t + \frac{1}{t}\right)^2 = 5$

length of focal chord = $a\left(t + \frac{1}{t}\right)^2$ = 5a

*52. If chord PQ subtends an angle θ at the vertex of $y^2 = 4ax$, then $\tan\theta =$

(A) $\frac{2}{3}\sqrt{7}$ (B) $\frac{-2}{3}\sqrt{7}$

(C) $\frac{2}{3}\sqrt{5}$ (D) $\frac{-2}{3}\sqrt{5}$

Sol. **(D)**

Angle made by chord PQ at vertex (0, 0) is given by

$$\tan\theta = \left(\frac{\frac{2}{t}+2t}{1-4}\right) = \frac{2\left(\frac{1}{t}+t\right)}{-3} = \frac{-2}{3}\sqrt{5}$$

Paragraph for Questions 53 and 54

Let $S = S_1 \cap S_2 \cap S_3$, where

$S_1 = \{z \in C : |z| < 4\}$, $S_2 = \left\{z \in C : \text{Im}\left[\frac{z-1+\sqrt{3}i}{1-\sqrt{3}i}\right] > 0\right\}$ and $S_3 = \{z \in C : \text{Re } Z > 0\}$.

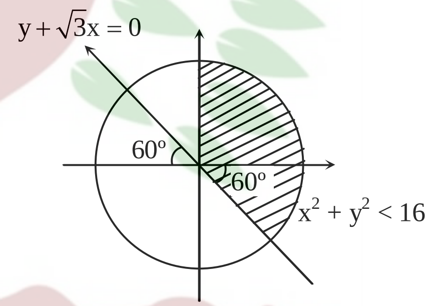
*53. Area of S =

(A) $\frac{10\pi}{3}$ (B) $\frac{20\pi}{3}$

(C) $\frac{16\pi}{3}$ (D) $\frac{32\pi}{3}$

Sol. **(B)**

Area of region $S_1 \cap S_2 \cap S_3$ = shaded area



$$= \frac{\pi \times 4^2}{4} + \frac{4^2 \times \pi}{6}$$

$$= 4^2\pi\left\{\frac{1}{4}+\frac{1}{6}\right\}$$

$$= \frac{20\pi}{3}$$

*54. $\min\limits_{z\in S}|1-3i-z| =$

(A) $\frac{2-\sqrt{3}}{2}$ (B) $\frac{2+\sqrt{3}}{2}$

(C) $\frac{3-\sqrt{3}}{2}$ (D) $\frac{3+\sqrt{3}}{2}$

Sol. **(C)**

Distance of (1, –3) from $y+\sqrt{3}x = 0$

$$> \left|\frac{-3+\sqrt{3}\times 1}{2}\right|$$

$$> \frac{3-\sqrt{3}}{2}$$

Paragraph for Questions 55 and 56

A box B_1 contains 1 white ball, 3 red balls and 2 black balls. Another box B_2 contains 2 white balls, 3 red balls and 4 black balls. A third box B_3 contains 3 white balls, 4 red balls and 5 black balls.

55. If 1 ball is drawn from each of the boxes B_1, B_2 and B_3, the probability that all 3 drawn balls are of the same colour is

(A) $\frac{82}{648}$ (B) $\frac{90}{648}$

(C) $\frac{558}{648}$ (D) $\frac{566}{648}$

Sol. **(A)**

P (required) = P (all are white) + P (all are red) + P (all are black)

$= \frac{1}{6}\times\frac{2}{9}\times\frac{3}{12}+\frac{3}{6}\times\frac{3}{9}\times\frac{4}{12}+\frac{2}{6}\times\frac{4}{9}\times\frac{5}{12}$

$= \frac{6}{648}+\frac{36}{648}+\frac{40}{648}=\frac{82}{648}$.

56. If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red, the probability that these 2 balls are drawn from box B_2 is

(A) $\frac{116}{181}$ (B) $\frac{126}{181}$

(C) $\frac{65}{181}$ (D) $\frac{55}{181}$

Sol. **(D)**

Let A : one ball is white and other is red

E_1 : both balls are from box B_1

E_2 : both balls are from box B_2

E_3 : both balls are from box B_3

Here, P (required) = $P\left(\frac{E_2}{A}\right)$

$$= \frac{P\left(\frac{A}{E_2}\right)\cdot P(E_2)}{P\left(\frac{A}{E_1}\right)\cdot P(E_1)+P\left(\frac{A}{E_2}\right)\cdot P(E_2)+P\left(\frac{A}{E_3}\right)\cdot P(E_3)}$$

$$= \frac{\frac{{}^2C_1\times{}^3C_1}{{}^9C_2}\times\frac{1}{3}}{\frac{{}^1C_1\times{}^3C_1}{{}^6C_2}\times\frac{1}{3}+\frac{{}^2C_1\times{}^3C_1}{{}^9C_2}\times\frac{1}{3}+\frac{{}^3C_1\times{}^4C_1}{{}^{12}C_2}\times\frac{1}{3}} = \frac{\frac{1}{6}}{\frac{1}{5}+\frac{1}{6}+\frac{2}{11}} = \frac{55}{181}.$$

SECTION – 3 : (Matching list Type)

This section contains 4 multiple choice questions. Each question has matching lists. The codes for the lists have choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

*57. Match List I with List II and select the correct answer using the code given below the lists :

	List – I		List – II
P.	$\left(\frac{1}{y^2}\left(\frac{\cos\left(\tan^{-1}y\right)+y\sin\left(\tan^{-1}y\right)}{\cot\left(\sin^{-1}y\right)+\tan\left(\sin^{-1}y\right)}\right)+y^4\right)^{1/2}$ takes value	1.	$\frac{1}{2}\sqrt{\frac{5}{3}}$
Q.	If cosx + cosy + cosz = 0 = sinx + siny + sinz then possible value of $\cos\frac{x-y}{2}$ is	2.	$\sqrt{2}$
R.	If $\cos\left(\frac{\pi}{4}-x\right)$cos2x + sinx sin2x secx = cosx sin2x secx + $\cos\left(\frac{\pi}{4}+x\right)$cos2x then possible value of secx is	3.	$\frac{1}{2}$
S.	If $\cot\left(\sin^{-1}\sqrt{1-x^2}\right)=\sin\left(\tan^{-1}\left(x\sqrt{6}\right)\right)$, x ≠ 0, then possible value of x is	4.	1

Codes :

	P	Q	R	S
(A)	4	3	1	2
(B)	4	3	2	1
(C)	3	4	2	1
(D)	3	4	1	2

Sol. **(B)**

$$P \to \frac{\cos\left(\tan^{-1}y\right)+y\sin\left(\tan^{-1}y\right)}{\cot\left(\sin^{-1}y\right)+\tan\left(\sin^{-1}y\right)}$$

$$=\frac{\frac{1}{\sqrt{1+y^2}}+\frac{y^2}{\sqrt{1+y^2}}}{\frac{\sqrt{1-y^2}}{y}+\frac{y}{\sqrt{1-y^2}}}=\frac{\sqrt{1+y^2}}{\frac{1}{y\sqrt{1-y^2}}}=y\sqrt{1-y^4}$$

$$\Rightarrow \frac{1}{y^2}\left(\frac{\cos\left(\tan^{-1}y\right)+y\sin\left(\tan^{-1}y\right)}{\cot\left(\sin^{-1}y\right)+\tan\left(\sin^{-1}y\right)}\right)^2+y^4$$

$$=\frac{1}{y^2}\left(y^2\left(1-y^4\right)\right)+y^4=1-y^4+y^4=1$$

Q → cos x + cos y + cos z = 0

sin x + sin y + sin z = 0

cos x + cos y = – cos z ..... (1)

sin x + sin y = – sin z ..... (2)

$(1)^2 + (2)^2$

1 + 1 + 2(cos x cos y + sin x sin y) = 1

2 + 2 cos (x – y) = 1

2 cos (x – y) = –1

$\cos(x-y) = -\frac{1}{2}$

$2\cos^2\left(\frac{x-y}{2}\right)-1=-\frac{1}{2}$

$2\cos^2\left(\frac{x-y}{2}\right)=\frac{1}{2}$

$$\cos^2\left(\frac{x-y}{2}\right)=\frac{1}{4}$$

$$\cos\left(\frac{x-y}{2}\right)=\frac{1}{2}$$

$R \to \cos\left(\frac{\pi}{4}-x\right)\cos 2x+\sin x\sin 2x\sec x$

$$= \cos x\sin 2x\sec x+\cos\left(\frac{\pi}{4}+x\right)\cos 2x$$

$$\left[\cos\left(\frac{\pi}{4}-x\right)-\cos\left(\frac{\pi}{4}+x\right)\right]\cos 2x = (\cos x \sin 2x - \sin x \sin 2x) \sec x$$

$$\frac{2}{\sqrt{2}}\sin x\cos 2x=(\cos x-\sin x)\sin 2x\sec x$$

$$\sqrt{2}\sin x\cos 2x=(\cos x-\sin x)2\sin x$$

$$\frac{1}{\sqrt{2}}=\frac{1}{\cos x+\sin x} \Rightarrow x=\frac{\pi}{4}$$

$$\sec x=\sec\frac{\pi}{4}=\sqrt{2}$$

$S \to \cot\left(\sin^{-1}\sqrt{1-x^2}\right)$

$$\cot\alpha=\frac{x}{\sqrt{1-x^2}}$$

$$\tan^{-1}\left(x\sqrt{6}\right)=\phi$$

$$\sin\phi=\frac{x\sqrt{6}}{\sqrt{6x^2+1}}$$

$$\Rightarrow \frac{x}{\sqrt{1-x^2}}=\frac{x\sqrt{6}}{\sqrt{6x^2+1}}$$

$$6x^2+1=6-6x^2$$

$$12x^2=5$$

$$x=\sqrt{\frac{5}{12}}=\frac{1}{2}\sqrt{\frac{5}{3}}$$

*58. A line L : $y = mx + 3$ meets y-axis at E(0, 3) and the arc of the parabola $y^2 = 16x$, $0 \le y \le 6$ at the point $F(x_0, y_0)$. The tangent to the parabola at $F(x_0, y_0)$ intersects the y-axis at $G(0, y_1)$. The slope m of the line L is chosen such that the area of the triangle EFG has a local maximum.

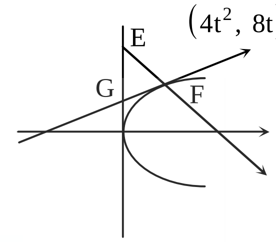
Match List I with List II and select the correct answer using the code given below the lists :

List – I		List – II	
P.	m =	1.	$\frac{1}{2}$
Q.	Maximum area of ΔEFG is	2.	4
R.	y_0 =	3.	2
S.	y_1 =	4.	1

Codes :

	P	Q	R	S
(A)	4	1	2	3
(B)	3	4	1	2
(C)	1	3	2	4
(D)	1	3	4	2

Sol. **(A)**



$A(t) = 2t^2(3 - 4t)$

For max. A(t), $t = \frac{1}{2}$

$\Rightarrow m = 1$

$\Rightarrow A(t)\big|_{max.} = \frac{1}{2}$ sq. units

$y_0 = 4$ and $y_1 = 2$

59. Match List I with List II and select the correct answer using the code given below the lists :

List – I		List – II	
P.	Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 2. Then the volume of the parallelepiped determined by vectors $2(\vec{a}\times\vec{b}), 3(\vec{b}\times\vec{c})$ and $(\vec{c}\times\vec{a})$ is	1.	100
Q.	Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 5. Then the volume of the parallelepiped determined by vectors $3(\vec{a}+\vec{b}), (\vec{b}+\vec{c})$ and $2(\vec{c}+\vec{a})$ is	2.	30
R.	Area of a triangle with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 20. Then the area of the triangle with adjacent sides determined by vectors $(2\vec{a}+3\vec{b})$ and $(\vec{a}-\vec{b})$ is	3.	24
S.	Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then the area of the parallelogram with adjacent sides determined by vectors $(\vec{a}+\vec{b})$ and $\vec{a}$ is	4.	60

Codes :

	P	Q	R	S
(A)	4	2	3	1
(B)	2	3	1	4
(C)	3	4	1	2
(D)	1	4	3	2

Sol. **(C)**

$P \rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right] = 2$

$$\left[2\vec{a}\times\vec{b}\ \ 3\vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right] = 6\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2 = 6\times4 = 24$$

$Q \rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right] = 5$

$$6\left[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}\right] = 12\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 60$$

$R \rightarrow \frac{1}{2}\left|\vec{a}\times\vec{b}\right| = 20$

$$\frac{1}{2}\left|(2\vec{a}+3\vec{b})\times(\vec{a}-\vec{b})\right|$$

$\frac{1}{2}\left|-2(\vec{a}\times\vec{b})-3(\vec{a}\times\vec{b})\right|$

$\frac{5}{2}\times 40=100$

S $\to$ $|\vec{a}\times\vec{b}|=30$

$\Rightarrow \left|(\vec{a}+\vec{b})\times\vec{a}\right| = |\vec{b}\times\vec{a}|=30$

60. Consider the lines $L_1 : \frac{x-1}{2}=\frac{y}{-1}=\frac{z+3}{1}$, $L_2 : \frac{x-4}{1}=\frac{y+3}{1}=\frac{z+3}{2}$ and the planes $P_1 : 7x + y + 2z = 3$, $P_2 : 3x + 5y - 6z = 4$. Let $ax + by + cz = d$ be the equation of the plane passing through the point of intersection of lines L_1 and L_2, and perpendicular to planes P_1 and P_2.

Match List I with List II and select the correct answer using the code given below the lists :

List – I		List – II	
P.	a =	1.	13
Q.	b =	2.	–3
R.	c =	3.	1
S.	d =	4.	–2

Codes :

	P	Q	R	S
(A)	3	2	4	1
(B)	1	3	4	2
(C)	3	2	1	4
(D)	2	4	1	3

Sol. **(A)**

Plane perpendicular to P_1 and P_2 has Direction Ratios of normal

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix} = -16\hat{i}+48\hat{j}+32\hat{k} \quad \dots (1)$$

For point of intersection of lines

$(2\lambda_1 + 1, -\lambda_1, \lambda_1 - 3) \equiv (\lambda_2 + 4, \lambda_2 - 3, 2\lambda_2 - 3)$

$\Rightarrow 2\lambda_1 + 1 = \lambda_2 + 4$ or $2\lambda_1 - \lambda_2 = 3$

$-\lambda_1 = \lambda_2 - 3$ or $\lambda_1 + \lambda_2 = 3$

$\Rightarrow \lambda_1 = 2, \lambda_2 = 1$

$\therefore$ Point is $(5, -2, -1)$ … (2)

From (1) and (2), required plane is

$-1(x-5) + 3(y+2) + 2(z+1) = 0$

or $-x + 3y + 2z = -13$

$x - 3y - 2z = 13$

$\Rightarrow a = 1, b = -3, c = -2, d = 13$.