IIT – JEE (2011) PAPER II QUESTION & SOLUTIONS CODE 0

PART I : CHEMISTRY

PAPER – II

SECTION – I(TOTAL MARKS: 24)

(Single Correct Answer Type)

This section contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. Among the following complexes (**K – P**),
$K_3[Fe(CN)_6]$ (**K**), $[Co(NH_3)_6]Cl_3$ (**L**), $Na_3[Co(oxalate)_3]$ (**M**), $[Ni(H_2O)_3]Cl_2$ (**N**), $K_2[Pt(CN)_4]$ (**O**) and $[Zn(H_2O)_6](NO_3)_2$ (**P**)
the diamagnetic complexes are

(A) **K, L, M, N** (B) **K, M, O, P**
(C) **L, M, O, P** (D) **L, M, N, O**

Key. : (C)

Sol. : $K_3Fe(CN)_6$ is paramagnetic
$[Ni(H_2O)_6]Cl_2$ is paramagnetic too.

2. Consider the following cell reaction :
$2Fe_{(s)} + O_{2(g)} + 4H^+_{(aq)} \rightarrow 2Fe^{2+}_{(aq)} + 2H_2O_{(l)}$ E° = 1.67 V
At $[Fe^{2+}] = 10^{-3}$M, $P(O_2) = 0.1$ atm and pH = 3, the cell potential at 25°C is

(A) 1.47 V (B) 1.77 V
(C) 1.87 V (D) 1.57 V

Key. : (D)

Sol. : $2Fe_{(s)} + O_{2(g)} + 4H^+ \longrightarrow 2Fe^{+2}_{(aq)} + 2H_2O(\ell)$ E° = 1.67 V

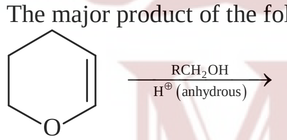
$$E_{cell} = E^{\circ}_{cell} - \frac{0.0591}{n}\log\frac{\left[Fe^{+2}\right]^2}{[O_2]\times\left[H^+\right]^4}$$

$$= 1.67 - \frac{0.0591}{4}\log\frac{\left[10^{-3}\right]^2}{\left(10^{-1}\right)\left(10^{-3}\right)^4}$$

$$= 1.67 - \frac{0.0591}{4}\log\frac{10^{-6}}{10^{-13}}$$

$$= 1.67 - \frac{0.0591}{4}\times 7 = 1.566 \approx 1.57 \text{ Volt.}$$

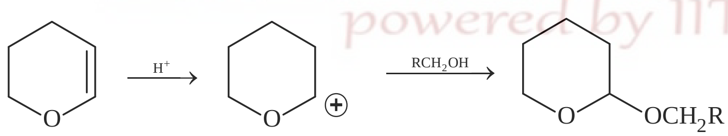
3. The major product of the following reaction is



(A) a hemiacetal (B) an acetal
(C) an ether (D) an ester

Key. : (B)

Sol. :



Acetal

4. Passing H_2S gas into a mixture of Mn^{2+}, Ni^{2+}, Cu^{2+} and Hg^{2+} ions in an acidified aqueous solution precipitates

(A) CuS and HgS (B) MnS and CuS

(C) MnS and NiS (D) NiS and HgS

Key.: (A)

Sol. : Cu^{2+} & Hg^{2+} ions belong to Group – II of salt analysis & their sulphides are very less soluble. Hence CuS and HgS will be precipitated out even in acidic medium by passing H_2S gas.

5. Oxidation states of the metal in the minerals haematite and magnetite, respectively are

(A) II, III and haematite and III in magnetite (B) II, III in haematite and II in magnetite

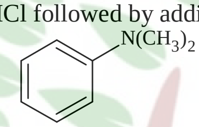
(C) II in haematite and II, III in magnetite (D) III in haematite and II, III in magnetite

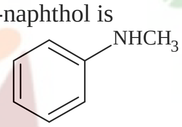
Key. : (C)

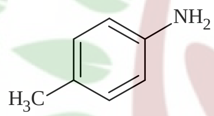
Sol. :

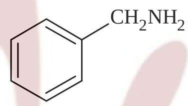
	Formula	Oxidation state of Fe
Haematite $\rightarrow$	Fe_2O_3	+3
Magnetile $\rightarrow$	Fe_3O_4 ($FeO + Fe_2O_3$)	+2, +3

6. Amongst the compounds given, the one that would from a brilliant colored dye on treatment with $NaNO_2$ in dil. HCl followed by addition to an alkaline solution of β-naphthol is

(A) 

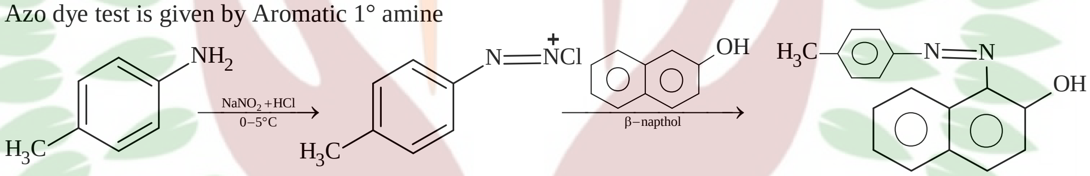
(B) 

(C) 

(D) 

Key. : (C)

Sol. : Azo dye test is given by Aromatic 1° amine



Red azo dye

7. The freezing point (in °C) of a solution containing 0.1 g of $K_3[Fe(CN)_6]$ (Mol. Wt. 329) in 100 g of water (K_f = 1.86 K kg mol^{-1}) is

(A) -2.3×10^{-2} (B) -5.7×10^{-2}

(C) -5.7×10^{-2} (D) -1.2×10^{-2}

Key: (A)

Sol.: $\Delta T_f = i \times k_f \times m$

$$= i \times k_f \times \frac{W_B}{M_B} \times \frac{1000}{W_A}$$

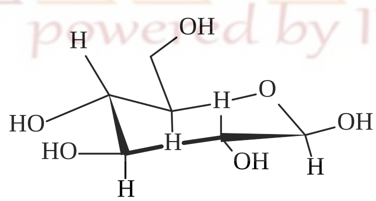
$$= 4 \times 1.86 \times \frac{0.1}{329} \times \frac{1000}{100}$$

$= 0.0226$

$= 2.26 \times 10^{-2} \approx 2.3 \times 10^{-2}$

Freezing point $= -2.3 \times 10^{-2}$.

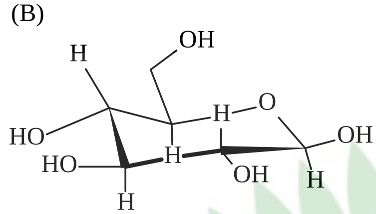
8. The following carbohydrate is



(A) a ketohexose
(B) an aldohexose
(C) an α-furanose
(D) an α - pyransose

Key. : (B)

Sol.



is an aldohexose.

SECTION – II (TOTAL MARKS: 16)
(Multiple Correct Answer(s) Type)

This section contains 4 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE may be correct.

9. The equilibrium

$$2Cu^{I} \rightleftharpoons Cu^{\circ} + Cu^{II}$$

In aqueous medium at 25°C shifts towards the left in the presence of

(A) NO_3^-
(B) Cl^-
(C) SCN^-
(D) CN^-

Key. : (B, C, D)

Sol. : The equilibrium constant for the reaction

$2Cu^+ \rightleftharpoons Cu + Cu^{2+}$ is high enough to keep it largely towards right. The Cu^+ will exist in water with stability when either present as complexes or as insoluble salt CuCl, CuCn and CuSCN are insoluble cuprous salt.

10. The correct functional group X and the reagent / reaction condition Y in the following scheme are

$$X-(CH_2)_4-X \xrightarrow[\text{(ii) } HO-\overset{O}{\overset{\|}{C}}-(CH_2)_4-\overset{O}{\overset{\|}{C}}-OH \text{, heat}]{\text{(i) Y}} \text{condensation polymer}$$

(A) X = $COOCH_3$, Y = H_2 / Ni / heat
(B) X = $CONH_2$, Y = H_2/ Ni / heat
(C) X = $CONH_2$, Y = Br_2 / NaOH
(D) X = CN, Y = H_2 / Ni/ heat

Key. : (A, B, C, D)

Sol. : (A)

$$H_3C-O-\overset{O}{\overset{\|}{C}}-(CH_2)_4-\overset{O}{\overset{\|}{C}}-OCH_3 \xrightarrow{H_2/Ni} CH_3OH + HO-CH_2-(CH_2)_4CH_2OH$$

(B)

$$H_2N-\overset{O}{\overset{\|}{C}}-(CH_2)_4-\overset{O}{\overset{\|}{C}}-NH_2 \xrightarrow{H_2/Ni} NH_2-(CH_2)_6-NH_2$$

(C)

$$H_2N-\overset{O}{\overset{\|}{C}}-(CH_2)_4-\overset{O}{\overset{\|}{C}}-NH_2 \xrightarrow{Br_2/NaOH} NH_2-(CH_2)_4-NH_2$$

(D)

$$NC-(CH_2)_4-CN \xrightarrow{H_2/Ni} NH_2-(CH_2)_6-NH_2$$

Diamines give polyamides with dicarboxylic acids
Similarly di-ols give polyester with dicarboxylic acid.

11. For the first order reaction

$2N_2O_2\,(g) \longrightarrow 4NO_2\,(g) + O_2\,(g)$

(A) the concentration of the reactant decreases exponentially with time.

(B) the half-life of the reaction decreases with increasing temperature.

(C) the half-life of the reaction depends on the initial concentration of the reactant.

(D) the reaction proceeds to 99.6% completion in eight half-life diration.

Key. : (A, B, D)

Sol. : First order reaction : $-\dfrac{d[N_2O_5]}{dt} = K.[N_2O_5]$

Half life period : $t_{1/2} = \dfrac{\ell n2}{K}$

With temperature 'K' increases and therefore $t_{1/2}$ decreases

At the completion of 8 half lives

Remaining % = $\dfrac{1}{28}\times 100 = \dfrac{1}{256}\times 100 = 0.39$

$\therefore$ % reacted = 100 – 0.39 = 99.61 %.

12. Reduction of the metal centre in aqueous permagnate ion involves

(A) 3 electrons in neutral medium

(B) 5 electrons in neutral medium

(C) 3 electrons in alkaline medium

(D) 5 electrons in acidic medium

Key. : (A, C, D)

Sol. : $8H^+ + 5e^- + MnO_4^- \longrightarrow Mn^{2+} + 4H_2O$

$2H_2O + 3e^- + MnO_4^- \longrightarrow MnO_2 + 4OH^-$

$2H_2O + 3e^- + MnO_4^- \longrightarrow MnO_2 + 4OH^-$

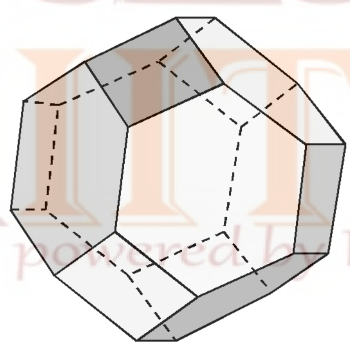
SECTION – III (TOTAL MARKS: 24)
(Integer Answer Type)

This Section contains 6 questions. The answer to each question is a Single-digit integer, ranging from 0 to 9. The bubble corresponding to the correct answer is to darkened in the ORS.

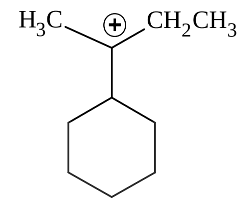
13. The number of hexagonal faces that are present in a truncated octahedron is

Key. : **6**

Sol. : The truncated octahedron is an Archimedean solid with 14 faces out of which 8 faces are hexagonal and rest 6 faces are square shaped.

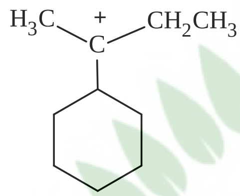


14. The total number of contributing structures showing hyperconjugation (involving C – H bonds) for the following carbocation is



Key. : 6

Sol. :



αH = 6

$\therefore$ contributing hyper conjugating structure = 6

15. Among the following, the number of compounds than can react with PCl_5 to give $POCl_3$ is O_2, CO_2, SO_2, H_2O, H_2SO_4, P_4O_{10}

Key. : 5

Sol. : $CO_2 + PCl_5 \longrightarrow POCl_3 + COCl_2$

$PCl_5 + H_2O \longrightarrow POCl_3 + 2\,HCl$

(if equi molar amounts are used, the reaction is more gentle and yields $POCl_3$)

$PCl_5 + SO_2 \longrightarrow POCl_3 + SOCl_2$

$PCl_5 + P_4O_{10} \longrightarrow 10\,POCl_3$

$PCl_5 + H_2SO_4 \longrightarrow POCl_3 + \underset{\text{chlorosulphonic acid}}{HSO_2Cl} + HCl$

$2PCl_5 + H_2SO_4 \longrightarrow \underset{\text{sulphuryl chloride}}{SO_2\,Cl_2} + 2POCl_3 + 2HCl$

16. In 1 L saturated solution of AgCl [K_{sp} (AgCl) = 1.6×10^{-10}], 0.1 mole of CuCl [K_{sp} (CuCl) = 1.0×10^{-6}] is added. The resultant concentration of Ag^+ in the solution is 1.6×10^{-x}. The value of "x" is

Key. : 7

Sol. : $AgCl(s) \rightleftharpoons \underset{x}{Ag^+} + \underset{x}{Cl^-}$

$CuCl(s) \rightleftharpoons \underset{y}{Cu^+} + \underset{y}{Cl^-}$

Net Cl^- in solution = (x + y)

k_{sp} AgCl = $1.6 \times 10^{-10} = [Ag^+]\,[Cl^-]$

$= x\,(x + y)$

$x(x + y) = 1.6 \times 10^{-10}$ . . . . .(i)

k_{sp} of CuCl = $1.0 \times 10^{-6} = [Cu^+][Cl^-]$

$1.0 \times 10^{-6} = y\,(x + y)$ . . . . (ii)

from equation (i) and (ii)

$$\frac{x}{y} = 1.6 \times 10^{-4}$$

$$y = \frac{10^4}{1.6}x$$

putting the value of y in eq. (i)

$$x.\left(x + \frac{10^4}{1.6}x\right) = 1.6 \times 10^{-10}$$

$$x^2 \times \frac{10^4}{1.6} = 1.6 \times 10^{-10}$$
$x^2 = (1.6)^2 \times 10^{-14}$
$x = 1.6 \times 10^{-7}$

17. The volume (in mL) of 0.1 M $AgNO_3$ required for complete precipitation of chloride ions present in 30 mL of 0.01 M solution of $[Cr(H_2O)_5Cl]Cl_2$, as silver chloride is close to

Key. : 6

Sol. : $2AgNO_3 + [Cr(H_2O)_5Cl]Cl_2 \longrightarrow 2AgCl + [Cr(H_2O)_5Cl](NO_3)_2$

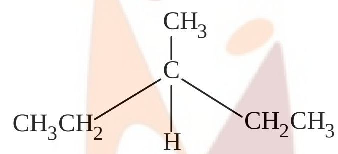
$\therefore$ milli equivalent . of $AgNO_3$ reacted = milli equivalent of $[Cr(H_2O)_5Cl]Cl_2$ reacted

thus $(M \times n \times V)_{AgNO_3} = (M \times n \times V)_{[Cr(H_2O)_5Cl]Cl_2}$

$\Rightarrow$ $0.1 \times 1 \times V = 0.01 \times 2 \times 30$

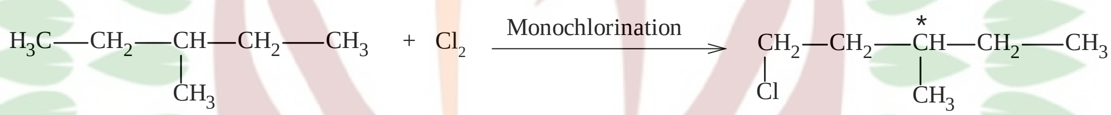
$\therefore$ $V = 6$

18. The maximum number of isomers (including stereoisomers) that are possible on mono-chlorination of the following compound, is



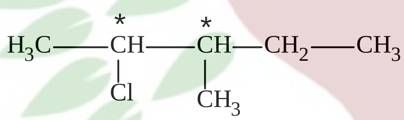
Key. : 8

Sol.



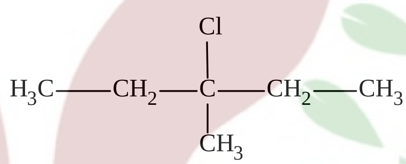
(1 chiral)

(A)



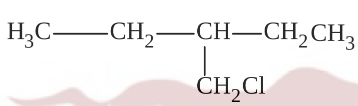
(2 chiral carbon)

(B)



(no chiral carbon)

(C)



(no chiral carbon)

(D)

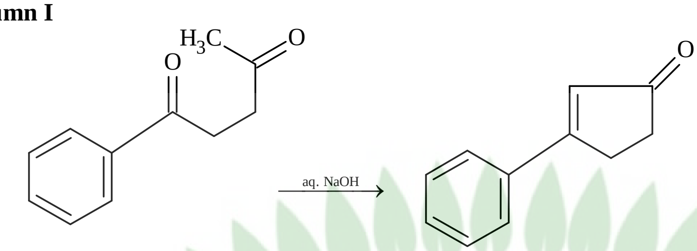
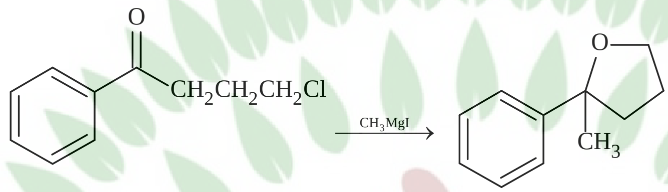
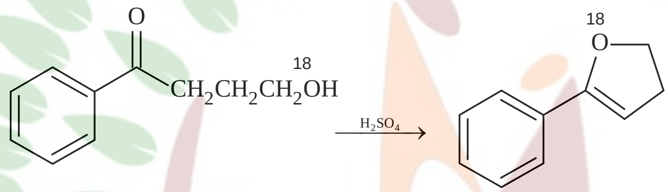
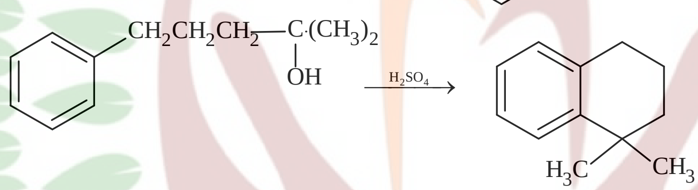
Thus 'A' has two structures, 'B' has 4, 'C' has 1, and 'D' has one therefore total product will be 8

SECTION – IV (TOTAL MARKS: 16)

(Matrix-Match Type)

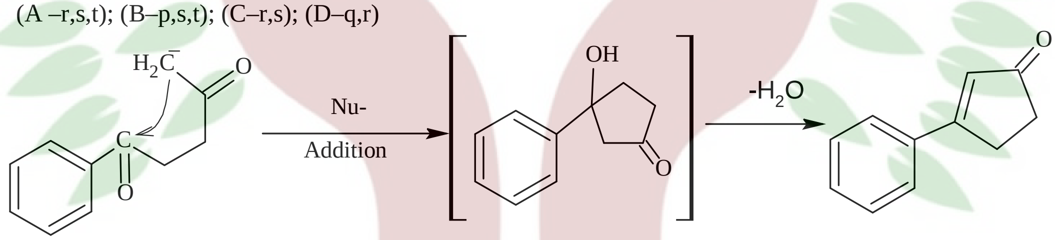
This section contains 2 questions. Each question four statements (A, B, C and D) given in Column I and five statements (p, q, r, s and t) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

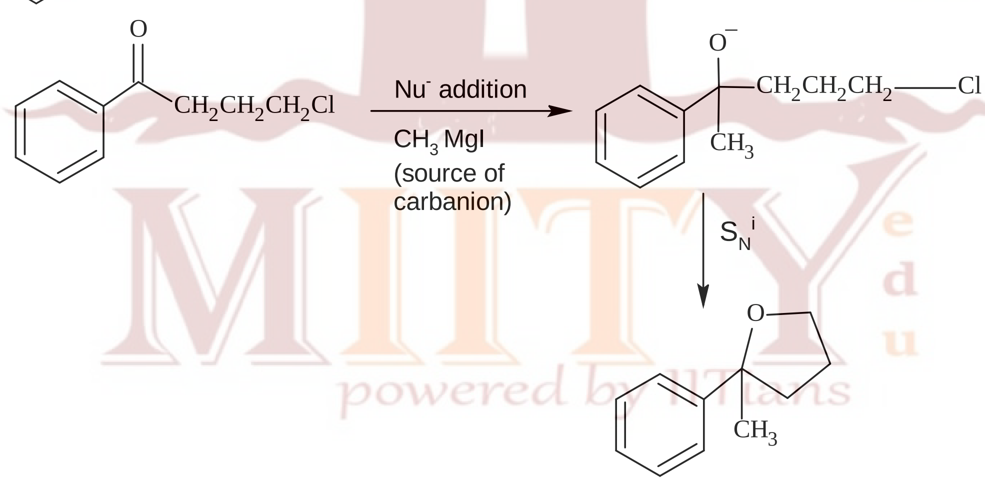
19. Match the reactions in Column I with appropriate types of steps / reactive intermediate involved in these reactions as given in Column II

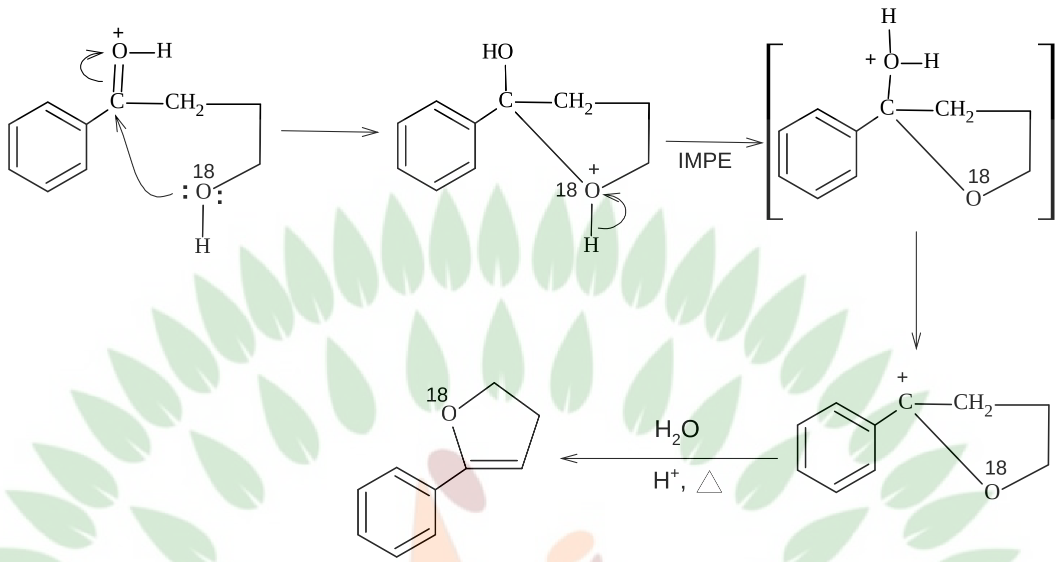
Column I		Column II	
(A)		(p)	Nucleophilic substitution
(B)		(q)	Electrophilic substitution
(C)		(r)	Dehydration
(D)		(s)	Nucleophilic
		(t)	Carbanion

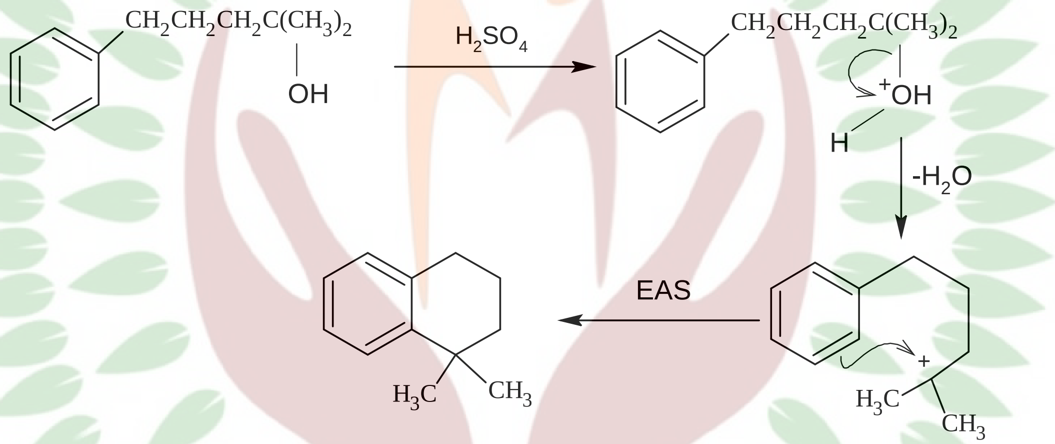
Key: (A –r,s,t); (B–p,s,t); (C–r,s); (D–q,r)

Sol.









20. Match the transformations in Column I with appropriate options in column II

Column I		Column II	
(A)	CO_2 (s) $\rightarrow$ CO_2 (g)	(p)	Phase transition
(B)	$CaCO_3$ (s) $\rightarrow$ CaO (s) + CO_2 (g)	(q)	Allotropic change
(C)	2H• $\rightarrow$ H_2 (g)	(r)	ΔH is positive
(D)	$P_{(\text{white, solid})} \rightarrow P_{(\text{red, solid})}$	(s)	ΔS is positive
		(t)	ΔS is negative

Key: (A –p,r,s); (B–r,s); (C–t); (D–q,r,t)

Sol.: (A) $CO_2(s) + Q \longrightarrow CO_2\ (g)$

ΔH = +ve , phase transition and ΔS = +ve

(B) $CaCO_3(s) + Q \longrightarrow CaO\ (s) + CO_2(g)$

ΔH = +ve, ΔS = +ve

(C) $2H^0 \longrightarrow H_2\ (g)$

two particle give one gaseous particle

ΔS = -ve

(D) $P_{\text{white , solid}} \xrightarrow{\Delta} P_{\text{red, solid}}$

ΔH = +ve, ΔS = +ve, allotropic change

PART II : PHYSICS

SECTION – I(TOTAL MARKS: 24)
(Single Correct Answer Type)

This section contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

21. A satellite is moving with a constant speed V in a circular orbit about the earth. An object of mass m is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is

(A) $\frac{1}{2}mV^2$ (B) mV^2

(C) $\frac{3}{2}mV^2$ (D) $2mV^2$.

Key: (B)

Sol.: At the time of ejection $E_{Total} = 0$

$\therefore$ Total Kinetic Energy = $E_{Total} - \text{P.E.} = \frac{GMm}{r}$

$= mV^2$

22. A point mass is subjected to two simultaneous sinusoidal displacements in x–direction, $x_1(t) = A\sin\omega t$ and $x_2(t) = A\sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B\sin(\omega t + \phi)$ brings the mass to a complete rest. The value of B and ϕ are

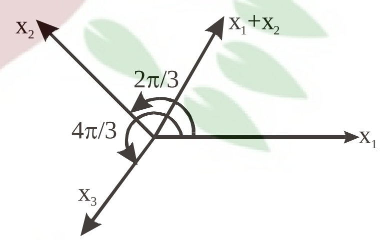
(A) $\sqrt{2}A, \frac{3\pi}{4}$ (B) $A, \frac{4\pi}{3}$

(C) $\sqrt{3}A, \frac{5\pi}{6}$ (D) $A, \frac{\pi}{3}$.

Key: (B)

Sol.: $x_1 + x_2 = A\sin(\omega t + \pi/3)$

$\because \quad x_1 + x_2 + x_3 = 0 \Rightarrow x_3 = A\sin\left(\omega t + \frac{4\pi}{3}\right)$



23. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

(A) 0.9% (B) 2.4%

(C) 3.1% (D) 4.2%.

Key: (C)

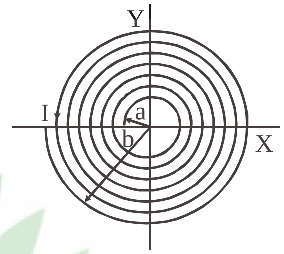
Sol.: Least count of screw gauge = $\frac{0.5\text{mm}}{50} = \frac{1}{100}$ mm.

$\therefore$ Diameter $D = 2.5 + 20 \times \frac{1}{100} = 2.7$ mm

Now, density $= \frac{m}{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3}$

$\therefore$ % error in density $= \frac{\Delta m}{m}\times 100 + 3\frac{\Delta D}{D}\times 100 = 3.1\%$.

24. A long insulated copper wire is closely wound as a spiral of N turns. The spiral has inner radius a and outer radius b. The spiral lies in the X–Y plane and a steady current I flows through the wire. The Z–component of the magnetic field at the centre of the spiral is



(A) $\frac{\mu_0 NI}{2(b-a)}\ell n\left(\frac{b}{a}\right)$ (B) $\frac{\mu_0 NI}{2(b-a)}\ell n\left(\frac{b+a}{b-a}\right)$

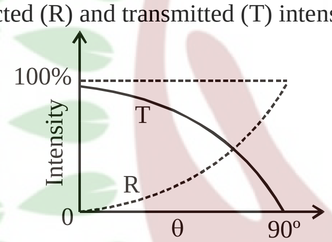
(C) $\frac{\mu_0 NI}{2b}\ell n\left(\frac{b}{a}\right)$ (D) $\frac{\mu_0 NI}{2b}\ell n\left(\frac{b+a}{b-a}\right)$.

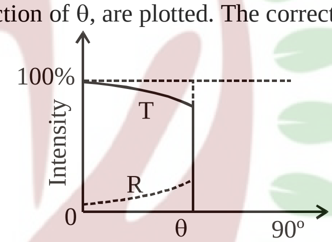
Key: (A)

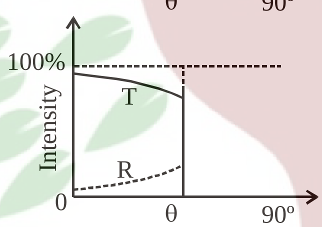
Sol.: $dN = \frac{N}{b-a}dr$

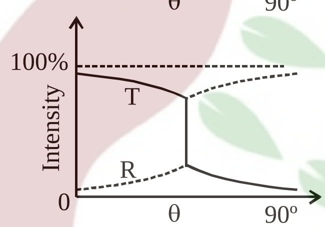
$$\therefore \quad B = \int_a^b \frac{\mu_0\left(\frac{N}{b-a}\right)Idr}{2r} = \frac{N\mu_0 I}{2(b-a)}\ln\left(\frac{b}{a}\right)$$

25. A light ray traveling in glass medium is incident on glass air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is

(A)


(B)


(C)


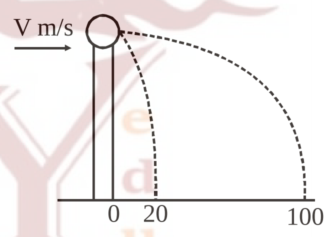
(D)


Key: (C)

Sol.: For $0 \le \theta < \theta_c$ there will be little reflection

For $\theta \ge \theta_c$ there will be no transmission.

26. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg traveling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity of the bullet is



(A) 250 m/s (B) $250\sqrt{2}$ m/s

(C) 400 m/s (D) 500 m/s.

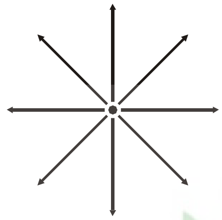
Key: (D)

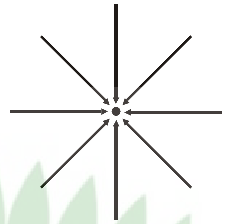
Sol.: $t = \sqrt{\frac{2\times 5}{10}} = 1s$

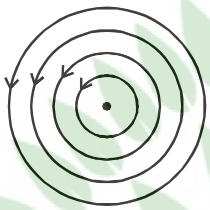
COM $\Rightarrow v\times 0.01 = 0.2\times 20 + 0.01\times 100$

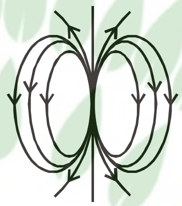
$\Rightarrow$ v = 500 m/s

27. Which of the field patterns given below is valid for electric field as well as for magnetic field ?

(A) 

(B) 

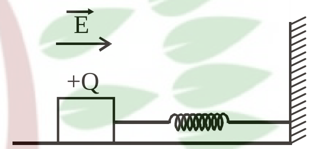
(C) 

(D) 

Key: (C)

Sol.: In case of non-conservative electric field, lines as shown in (C) are possible. Also, in case of toroid, magnetic field lines are circular

28. A wooden block performs SHM on a frictionless surface with frequency, v_0. The block carries a charge +Q on its surface. If now a uniform electrical field $\vec{E}$ is switched on as shown, then the SHM of the block will be



(A) of the same frequency and with shifted mean position
(B) of the same frequency and with the same mean position
(C) of changed frequency and with shifted mean position
(D) of changed frequency and with the same mean position.

Key: (A)

Sol.: An additional constant force will not change the frequency but will shift the mean position.

SECTION – II (TOTAL MARKS: 16)
(Multiple Correct Answer(s) Type)

This section contains 4 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE may be correct.

29. A series R–C circuit is connected to AC voltage source. Consider two cases; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true ?

(A) $I_R^A > I_R^B$ (B) $I_R^A < I_R^B$

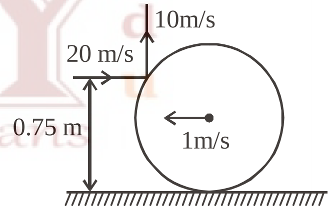
(C) $V_C^A > V_C^B$ (D) $V_C^A < V_C^B$.

Key: (B, C)

Sol.: $Z = \sqrt{R^2 + \dfrac{1}{\omega^2 c^2}}$

$\therefore \quad z_A > z_B \Rightarrow I_R^A < I_R^B$ and $v_C^A > v_C^B$

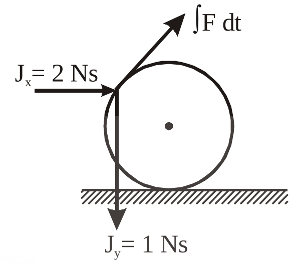
30. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction, hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision



(A) the ring has pure rotation about its stationary CM
(B) the ring comes to a complete stop
(C) friction between the ring and the ground is to the left
(D) there is no friction between the ring and the ground.

Key: (C)

Sol.: Assuming friction to be absent at point of contact (to check the tendency of sliding), we find that under the impulsive forces shown in figure, the point of contact moves to the right hence friction will act towards left.



31. Which of the following statement(s) is/are correct ?

(A) if the electric field due to a point charge varies as $r^{-2/5}$ instead of r^{-2}, then the Gauss law will still be valid

(B) the Gauss law can be used to calculate the field distribution around an electric dipole

(C) if the electric field between two point charges is zero somewhere, then the sign of the two charges is the same

(D) the work done by the external force in moving a unit positive charge from point A at potential V_A to point B at the potential V_B $(V_B - V_A)$.

Key: (C, D)

Sol.: If $E \propto r^{-2.5}$ then $\oint_{\text{sphere}} \vec{E}\cdot d\vec{s} = f(r) \neq \frac{Q_{enc}}{\varepsilon_0}$

$\therefore$ (A) is incorrect.

As field of an electric dipole is not symmetric, $\oint \vec{E}\cdot d\vec{s}$ is incalculable

$\therefore$ (B) is incorrect.

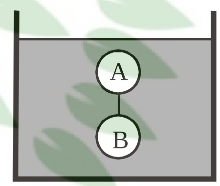
In case of similar charges, null point is between the two charges.

$\therefore$ (C) is correct.

$W_{ext} = q(V_B - V_A)$

$\therefore$ (D) is incorrect.

32. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if



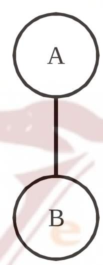
(A) $d_A < d_F$ (B) $d_B > d_F$

(C) $d_A > d_F$ (D) $d_A + d_B = 2\,d_F$.

Key: (A, B, D)

Sol.: For equilibrium of system

$V(d_A + d_B)g = 2V\,d_F\,g$

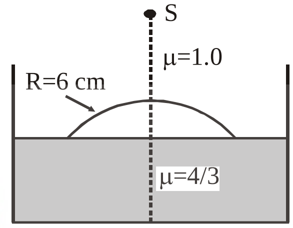


SECTION – III (TOTAL MARKS: 24)

(Integer Answer Type)

This Section contains 6 questions. The answer to each question is a Single-digit integer, ranging from 0 to 9. The bubble corresponding to the correct answer is to darkened in the ORS.

33. Water (with refractive index $=\frac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature R = 6 cm as shown. Consider oil to act as a thin lens. An object S is placed 24 cm above water surface. The location of its image is at x cm above the bottom of the tank. Then x is



Key: 2

Sol.: Refraction at air-oil boundary

$$\frac{7/4}{V_1}-\frac{1}{-24}=\frac{(7/4)-1}{+6}$$

$$\frac{7}{4V_1}=\frac{-1}{24}+\frac{1}{8}=\frac{-1+3}{24}=\frac{1}{12}$$

$$V_1=\frac{7\times12}{4}=21\text{cm}$$

Refraction at oil-water boundary

$$\frac{4/3}{V}-\frac{7/4}{+21}=0\Rightarrow\frac{4}{3V}=\frac{1}{12}$$

$$\frac{4}{3V}=\frac{1}{12}\Rightarrow V=\frac{4\times12}{3}=16\text{cm}$$

x = 2 cm

34. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free–space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the shphere is A × 10^Z (where 1 < A < 10). The value of Z is

Key: 7

Sol.: $$E=\frac{hc}{\lambda}=\frac{1242}{200}=6.21\text{eV}$$

$$V=\frac{1}{4\pi\varepsilon_0}\frac{ne}{r}$$

When emission will stop then

$$E=eV+\phi$$

$$\Rightarrow\ \frac{n\left(1.6\times10^{-19}\right)^2\times9\times10^9}{10^{-2}}=1.51\times\left(1.6\times10^{-19}\right)$$

$$\Rightarrow\ n=\frac{1.5\times10^{-2}}{1.6\times9\times10^{-10}}=\frac{1.51}{1.6\times9}\times10^8=\frac{15.1}{16\times9}\times10^7$$

$\Rightarrow$ z= 7

35. A train is moving along a straight line with a constant acceleration a. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60º to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2 is

Key: 5

Sol.: $$u_y=10\sin60=5\sqrt{3}\text{ m/s}$$

$$\Rightarrow\quad t=\frac{2u_y}{g}=\frac{2\times5\sqrt{3}}{10}=\sqrt{3}\text{s}$$

$$S_x = u_x t + \frac{1}{2}a_x t^2$$

$$1.15 = 5\times t - \frac{1}{2}a\times t^2$$

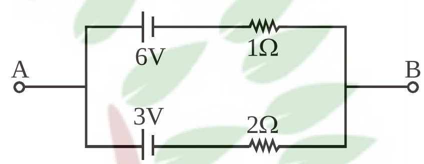
$$1.15 = 5\times\sqrt{3} - \frac{3}{2}a$$

$$\frac{3a}{2} = 5\times 1.73 - 1.15 = 8.65 - 1.15$$

$$\frac{3a}{2} = 7.5$$

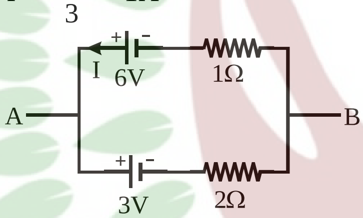
$$\Rightarrow\ a = \frac{15}{3} = 5\,\text{m/s}^2$$

36. Two batteries of different emfs and different internal resistances are connected as shown. The voltage across AB in volts is



Key: 5

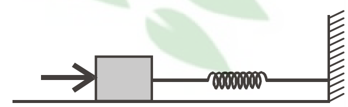
Sol.: $I = \dfrac{6-3}{3} = 1\text{A}$



$V_A - 3 - 2\times 1 = V_B$

$V_{AB} = 5\,V$

37. A block of mass 0.18 kg is attached to a spring of force–constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un–stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is V = N/10. Then N is



Key: 4

Sol.: Work done by friction

$= -\mu mgx = -[0.1\times 0.18\times 10\times 0.06] = -108\times 10^{-4}$

$\frac{1}{2}mv^2 - 108\times 10^{-4} = \frac{1}{2}kx^2 = \frac{1}{2}\times 2\times (0.06)^2$

$= 36\times 10^{-4}$

$0.09v^2 = 144\times 10^{-4}$

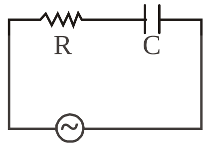
$v = \frac{12}{3}\times 10^{-1} = \frac{4}{10}$

N = 4

38. A series R–C combination is connected to an AC voltage of angular frequency ω = 500 radian/s. If the impedance of the R–C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is

Key: 4

Sol.: $Z = \sqrt{R^2 + X_C^2} = R\sqrt{\frac{5}{4}}$



$$R^2 + X_C^2 = \frac{5}{4}R^2$$

$$X_C = \frac{R}{2} = \frac{2}{\omega R}$$

$$C = \frac{2}{\omega R}$$

$$\tau = RC = R\frac{2}{\omega R} = \frac{2}{\omega}$$

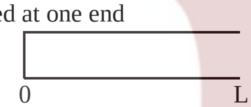
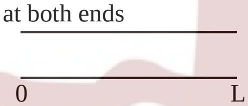
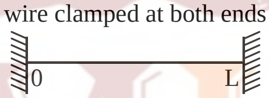
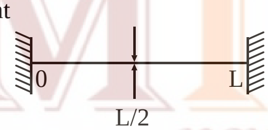
$$= \frac{2}{500} = 4 \times 10^{-3} = 4\text{ms}$$

SECTION – IV (TOTAL MARKS: 16)

(Matrix-Match Type)

This section contains 2 questions. Each question four statements (A, B, C and D) given in Column I and five statements (p, q, r, s and t) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

39. Column I shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency whose wavelength is denoted as λ_1. Match each system with statements given in Column II describing the nature and wavelength of the standing waves.

Column I	**Column II**
(A) Pipe closed at one end 	(p) Longitudinal waves
(B) Pipe open at both ends 	(q) Transverse waves
(C) Stretched wire clamped at both ends 	(r) $\lambda_f = L$
(D) Stretched wire clamped at both ends and at mid–point 	(s) $\lambda_f = 2L$
	(t) $\lambda_f = 4L$

Key: (A –p,t); (B–p,s); (C–q,s); (D–q,r)

Sol.: **(**A) $\frac{\lambda_1}{4} = L \Rightarrow \lambda_1 = 4L$

Longitudinal wave A – p, t

(B) $\frac{\lambda_1}{2} = L \Rightarrow \lambda_1 = 2L$

Longitudinal wave B – p, s

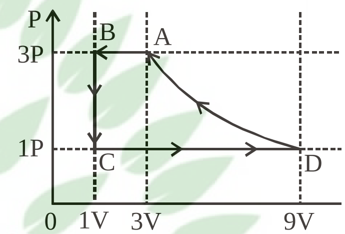
(C) $\frac{\lambda_1}{2} = L \Rightarrow \lambda_1 = 2L$

Transverse wave C- q, s

(D) $\frac{\lambda_1}{2} + \frac{\lambda_1}{2} = L \Rightarrow \lambda_1 = L$

Transverse wave D- q, r

40. One mole of a monatomic ideal gas is taken through a cycle ABCDA as shown in the P–V diagram. Column II gives the characteristics involved in the cycle. Match them with each of the processes given in Column I.



Column I		Column II	
(A)	Process A – B	(p)	Internal energy decreases
(B)	Process B – C	(q)	Internal energy increases
(C)	Process C – D	(r)	Heat is lost
(D)	Process D – A	(s)	Heat is gained
		(t)	Work is done on the gas.

Key: (A –p,r,t); (B–p,r); (C–q,s); (D–r,t)

Sol.: (A) Volume decreases

$\therefore$ Temperature decreases A- p, r, t

(B) Pressure decreases

$\therefore$ Temperature decreases B- p, r

(C) Volume increases

$\therefore$ Temperature increases C- q, s

(D) $T_D = \frac{PV}{nR} = \frac{(P)(9V)}{nR} = \frac{9PV}{nR}$

$T_A = \frac{(3P)(3V)}{nR} = \frac{9PV}{nR}$ D – r, t

PART I : MATHEMATICS

SECTION – I(TOTAL MARKS: 24)

(Single Correct Answer Type)

This section contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

41. A value of b for which the equations

$x^2 + bx - 1 = 0$

$x^2 + x + b = 0$

have one root in common is

(A) $-\sqrt{2}$ (B) $- i\sqrt{3}$

(C) $i\sqrt{5}$ (D) $\sqrt{2}$

Key: (B)

Sol.: $x^2 + bx - 1 = 0$ ... (i)

$x^2 + x + b = 0$ ... (ii)

(i) - (ii) gives $x = \frac{b+1}{b-1}$ which is the common root

Putting x in (ii)

$\left(\frac{b+1}{b-1}\right)^2 + \left(\frac{b+1}{b-1}\right) + b = 0$

$\Rightarrow b^3 + 3b = 0$

$b(b^2 + 3) = 0$

$b = 0, b^2 = -3$

$b = \pm\sqrt{3}\, i$

$b = -\sqrt{3}\, i.$

42. The circle passing through the point (-1, 0) and touching the y-axis at (0, 2) also passes through the point

(A) $\left(-\frac{3}{2}, 0\right)$ (B) $\left(-\frac{5}{2}, 2\right)$

(C) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (D) (-4, 0)

Key: (D)

Sol.: Equation of family of circle touching y-axis at (0, 2) will be

$(x - 0)^2 + (y - 2)^2 + \lambda x = 0$

Since this passes through, (–1, 0)

$1 + 4 - \lambda = 0$

$\lambda = 5$

Equation of circle is

$x^2 + y^2 + 5x - 4y + 4 = 0$

Also passes through (-4, 0).

43. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in R$. Then the set of all x satisfying (f o g o g o f) (x) = (g o g o f) (x), where (f o g) (x) = f(g(x)), is

(A) $\pm\sqrt{n\pi}$, $n \in \{0, 1, 2, ...\}$ (B) $\pm\sqrt{n\pi}$, $n \in \{1, 2, ...\}$

(C) $\frac{\pi}{2} + 2n\pi$, $n \in \{..., -2, -1, 0, 1, 2, ...\}$ (D) $2n\pi$, $n \in \{..., -2, -1, 0, 1, 2, ...\}$

Key: (A)

Sol.: $f(x) = x^2$, $g(x) = \sin x$

$g(f(x)) = g(x^2) = \sin x^2$

$g(g(f(x)) = \sin(\sin(x^2))$

$f.(g.(g.(f(x)))) = (\sin(\sin(x^2)))^2$

$f.(g.(g(f(x)))) = g.(g.(f(x))$

$(\sin(\sin(x^2)))^2 = \sin(\sin(x^2))$

$(\sin(\sin(x^2)))\,[\sin.\sin(x^2) - 1] = 0$

$\Rightarrow \sin(\sin(x^2)) = 0$ [or, $\sin(\sin(x^2)) \neq 1$) $\because \sin(x^2) \neq \frac{\pi}{2}$]

$\Rightarrow \sin(x^2) = 0$

$x = \pm\sqrt{n\pi}$ {n = 0, 1, 2, ... }

44. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$,

where each of a, b, and c is either ω or ω^2. Then the number of distinct matrices in the set S is

(A) 2 (B) 6

(C) 4 (D) 8

Key: (A)

Sol.: Let A = $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$

|A| = (1 - aω) (1 - cω)

For A to be non-singular matrix, none of a and c should be ω^2.

So, a = c = ω

While b can take value ω or ω^2

So, the number of distinct matrices in the set S is 2.

45. Let P (6, 3) be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the x-axis at (9, 0), then the eccentricity of the hyperbola is

(A) $\sqrt{\frac{5}{2}}$ (B) $\sqrt{\frac{3}{2}}$

(C) $\sqrt{2}$ (D) $\sqrt{3}$

Key: (B)

Sol.: Let P be (a secθ, btanθ)

a secθ = 6, b tanθ = 3

The equation of normal at P is

ax cosθ + bycotθ = $a^2 + b^2$

Put y = 0, x = $\frac{a^2+b^2}{a\cos\theta} = 9 \Rightarrow \frac{a^2+b^2}{a.\frac{a}{6}} = 9$

$\Rightarrow 1 + \frac{b^2}{a^2} = \frac{3}{2} \Rightarrow e = \sqrt{\frac{3}{2}}$

46. Let f : [-1, 2] → [0, ∞) be a continuous function such that f(x) = f(1-x) for all x∈ [-1, 2]. Let $R_1 = \int_{-1}^{2} xf(x)\,dx$, and R_2 be the area of the region bounded by y = f(x), x = -1, x = 2, and the x-axis. Then

(A) $R_1 = 2R_2$ (B) $R_1 = 3R_2$

(C) $2R_1 = R_2$ (D) $3R_1 = R_2$

Key: (C)

Sol.: $R_1 = \int_{-1}^{2} xf(x)dx$, $R_2 = \int_{-1}^{2} f(x)dx$

$R_1 = \int_{-1}^{2} (1-x)f(1-x)dx$ (replace x by (-1 + 2 - x))

$= \int_{-1}^{2} (1-x)f(x)dx = \int_{-1}^{2} f(x)dx - \int_{-1}^{2} xf(x)dx$

$\Rightarrow R_1 = R_2 - R_1 \Rightarrow 2R_1 = R_2$.

47. If $\lim_{x\to 0}\left[1 + x\ln(1+b^2)\right]^{\frac{1}{x}} = 2b\sin^2\theta$, b > 0 and θ∈ (-π, π], then the value of θ is

(A) $\pm\frac{\pi}{4}$ (B) $\pm\frac{\pi}{3}$

(C) $\pm\frac{\pi}{6}$ (D) $\pm\frac{\pi}{2}$

Key: (D)

Sol.: $\lim_{x\to 0}[1 + x\ln(1+b^2)]^{1/x} = 2b\sin^2\theta$ (b > 0)

$\Rightarrow e^{\lim_{x\to 0}\frac{1}{x}.[x\ln(1+b^2)]} = 2b\sin^2\theta$

$\Rightarrow 1 + b^2 = 2b \sin^2\theta$

$\Rightarrow \sin^2\theta = \frac{1}{2}(b + \frac{1}{b}) \ge 1$

$\Rightarrow \sin^2\theta = 1$ (when b = 1) $\Rightarrow \theta = \pm\frac{\pi}{2}$

48. Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from (0, 0) to (x, y) in the ratio 1 : 3. Then the locus of P is

(A) $x^2 = y$ (B) $y^2 = 2x$
(C) $y^2 = x$ (D) $x^2 = 2y$

Key: (C)

Sol.: Let P be (h, k). So, $h = \frac{x}{4}$ and $k = \frac{y}{4}$

As, $y^2 = 4x \Rightarrow 16k^2 = 16h \Rightarrow k^2 = h$

So, locus of P is $y^2 = x$

SECTION – II (TOTAL MARKS: 16)
(Multiple Correct Answer(s) Type)

This section contains 4 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE may be correct.

49. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If P(T) denotes the probability of occurrence of the event T, then

(A) $P(E) = \frac{4}{5}, P(F) = \frac{3}{5}$ (B) $P(E) = \frac{1}{5}, P(F) = \frac{2}{5}$

(C) $P(E) = \frac{2}{5}. P(F) = \frac{1}{5}$ (D) $P(E) = \frac{3}{5}, P(F) = \frac{4}{5}$

Key: (A, D)

Sol.: $P(E) + P(F) - 2P(E\cap F) = \frac{11}{25}$

$1 - [P(E) + P(F) - P(E \cap F)] = \frac{2}{25}$

$\Rightarrow P(E) + P(F) = \frac{7}{5}, P(E \cap F) = \frac{12}{25} \Rightarrow P(E) . P(F) = \frac{12}{25}$

$\Rightarrow P(E) = \frac{3}{5}, P(F) = \frac{4}{5}$ or $P(E) = \frac{4}{5}, P(F) = \frac{3}{5}$

50. Let $f : (0, 1) \to R$ be defined by $f(x) = \frac{b-x}{1-bx}$, where b is a constant such that $0 < b < 1$. Then

(A) f is not invertible on (0, 1) (B) $f \ne f^{-1}$ on (0, 1) and $f'(b) = \frac{1}{f'(0)}$

(C) $f = f^{-1}$ on (0, 1) and $f'(b) = \frac{1}{f'(0)}$ (D) f^{-1} is differentiable on (0, 1)

Key: (A)

Sol.: As range of $f(x) \ne R$

$\Rightarrow$ f(x) is not invertible.

51. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point (9, 6), then L is given by

(A) $y - x + 3 = 0$ (B) $y + 3x - 33 = 0$
(C) $y + x - 15 = 0$ (D) $y - 2x + 12 = 0$

Key: (A, B, D)

Sol.: Equation of normal is
$y + tx = 2t + t^3$
As it passes through (9, 6)
$6 + 9t = t^3 + 2t$
or, $t^3 - 7t - 6 = 0$
or, $(t + 1)(t^2 - t - 6) = 0$
or, $(t + 1)(t - 3)(t + 2) = 0$
$t = -1, -2, 3$
$\therefore$ equation of normals are
$y - x + 3 = 0$, $y - 2x + 12 = 0$
$y + 3x - 33 = 0$

52. If $f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \le -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2} < x \le 0 \\ x - 1, & 0 < x \le 1 \\ \ln x, & x > 1, \end{cases}$

then

(A) f(x) is continuous at $x = -\frac{\pi}{2}$

(B) f(x) is not differentiable at x = 0

(C) f(x) is differentiable at x = 1

(D) f(x) is differentiable at $x = -\frac{3}{2}$

Key: (A, B, C, D)

Sol.: $f\left(\frac{-\pi}{2}\right) = \lim_{x \to \frac{\pi^-}{2}} f(x) = \lim_{x \to \frac{-\pi^+}{2}} f(x) = 0$

$\therefore$ f(x) is continuous at $x = -\pi/2$
f(x) is also continuous at x = 0, 1

$$f'(x) = \begin{cases} -1 & x < \frac{-\pi}{2} \\ \sin x & -\frac{\pi}{2} < x < 0 \\ 1 & 0 < x < 1 \\ \frac{1}{x} & x > 1 \end{cases}$$

f(x) is not diff. at x = 0
f(x) is diff. at x = 1
f(x) is diff. at x = -3/2

SECTION – III (TOTAL MARKS: 24)

(Integer Answer Type)

This Section contains 6 questions. The answer to each question is a Single-digit integer, ranging from 0 to 9. The bubble corresponding to the correct answer is to darkened in the ORS.

53. Let M be a 3 × 3 matrix satisfying $M\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}-1\\2\\3\end{bmatrix}$, $M\begin{bmatrix}1\\-1\\0\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix}$, and $M\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\12\end{bmatrix}$. Then the sum of the diagonal entries of M is

Key: (9)

Sol.: $M = \begin{bmatrix} a & d & l \\ b & e & m \\ c & f & n \end{bmatrix}_{3\times3} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}_{3\times3} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

$d = -1$
$e = 2$
$f = 3$

$= \begin{bmatrix} a & d & l \\ b & e & m \\ c & f & n \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

$a - d = 1 \Rightarrow a = 0$
$b - e = 1 \Rightarrow b = 3$
$c - f = -1 \Rightarrow c = 2$

$= \begin{bmatrix} a & d & l \\ b & e & m \\ c & f & n \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$

$a + d + l = 0 \;\Rightarrow\; l = 1$
$b + e + m = 0 \;\Rightarrow m = -5$
$c + f + n = 12 \;\Rightarrow n = 7$

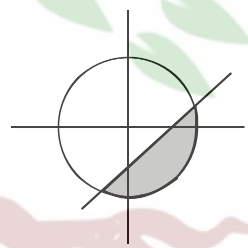
$M = \begin{bmatrix} 0 & -1 & 1 \\ 3 & 2 & -5 \\ 2 & 3 & 7 \end{bmatrix}$

Sum of diagonal entries = 9

54. The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \le 6$ into two parts. If $S = \left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\}$, then the number of point(s) in S lying inside the smaller part is

Key: (2)

Sol.: $x^2 + y^2 \le 6$
$L \equiv 2x - 3y - 1 > 0$
$S \equiv x^2 + y^2 - 6 < 0$
Clearly $\left(2, \frac{3}{4}\right)$ and $\left(\frac{1}{4}, -\frac{1}{4}\right)$ satisfies above two inequality



55. Let $\omega = e^{i\pi/3}$, and a, b, c, x, y, z be non-zero complex numbers such that
$a + b + c = x$
$a + b\omega + c\omega^2 = y$
$a + b\omega^2 + c\omega = z$. Then the value of $\dfrac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |C|^2}$ is

Sol.: Value is not fixed.

56. Let $y'(x) + y(x)\, g'(x) = g(x)\, g'(x)$, $y(0) = 0$, $x \in R$, where $f'(x)$ denotes $\dfrac{df(x)}{dx}$ and g(x) is a given non-constant differentiable function on R with $g(0) = g(2) = 0$. Then the value of y(2) is

Key: (0)

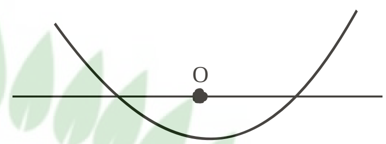
Sol.: I.F. $= e^{\int g'(x)dx} = e^{g(x)}$
$y(x)\, e^{g(x)} = \int e^{g(x)}.g(x)\, g'(x)dx = e^{g(x)}\,(g(x) - 1) + c$

$y(x) = (g(x) - 1) + ce^{-g(x)}$
Put $x = 0$, $0 = (-1) + c \Rightarrow c = 1$
$\therefore\ y(2) = (-1) + 1.e^{-0}$
$\therefore\ y(2) = 0.$

57. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is

Key: (2)

Sol.: Let $f(x) = x^4 - 4x^3 + 12x^2 + x - 1$
$f'(x) = 4x^3 - 12x^2 + 24x + 1$
$f''(x) = 12x^2 - 24x + 24$
Hence $f''(x) > 0\ \forall x \in R$
$f'(x)$ is strictly increasing function. So, $f(x)$ will have only one point of extrema.
$f(0) = -1$
Hence $f(x) = 0$ has two distinct real roots.



58. Let $a = -\hat{i} - \hat{k}$, $\vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$, then the value of $\vec{r}.\vec{b}$ is

Key: (9)

Sol.:
$$\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$$
$$\vec{a} \times (\vec{r} \times \vec{b}) = \vec{a} \times (\vec{c} \times \vec{b})$$
$$(\vec{a}.\vec{b})\vec{r} - (\vec{a}.\vec{r})\vec{b} = \vec{a} \times (\vec{c} \times \vec{b})$$
$$\vec{r} - 0 = \vec{a} \times (\vec{c} \times \vec{b})$$
$$\vec{r} = \vec{a} \times (\vec{c} \times \vec{b})$$
$$\vec{c} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 1 & 0 \end{vmatrix} = \hat{i}(0-3) + \hat{j}(-3-0) + \hat{k}(1+2) \ = -3\hat{i} - 3\hat{j} + 3\hat{k}$$
$$\vec{r} = \vec{a} \times (\vec{c} \times \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & -1 \\ -3 & -3 & 3 \end{vmatrix} = \hat{i}(0-3) + \hat{j}(3+3) + \hat{k}(3-0)$$
$$\vec{r} = -3\hat{i} + 6\hat{j} + 3\hat{k}$$
$$\therefore \quad \vec{r}.\vec{b} = (-3\hat{i} + 6\hat{j} + 3\hat{k}).(-\hat{i} + \hat{j}) \ = 3 + 6 = 9$$

SECTION – IV (TOTAL MARKS: 16)

(Matrix-Match Type)

This section contains 2 questions. Each question four statements (A, B, C and D) given in Column I and five statements (p, q, r, s and t) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

59. Match the statement given in Column I with the intervals/union of intervals given in Column II

	• Column I		• Column II
(A)	The set $\{Re\left(\frac{2iz}{1-z^2}\right)$: z is a complex number, $\|z\| = 1$, $z \neq \pm 1\}$	(p)	$(-\infty, -1) \cup (1, \infty)$
(B)	The domain of the function $f(x) = \sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is	(q)	$(-\infty, 0) \cup (0, \infty)$

(C) If $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$, then the set $\{f(\theta) : 0 \le \theta < \frac{\pi}{2}\}$ is | (r) $(-\infty, -1] \cup [1, \infty)$

(D) If $f(x) = x^{3/2}(3x - 10)$, $x \ge 0$, then f(x) is increasing in | (s) $(-\infty, 0] \cup [2, \infty)$

(t) $(-\infty, 0] \cup [2, \infty)$

Key: (A–s), (B–t), (C–r), (D–r)

Sol.: (A) $z = e^{i\theta}$

$$\text{Re}\left(\frac{2ie^{i\theta}}{1-e^{i2\theta}}\right) = \text{Re}\left(\frac{2ie^{i\theta}}{1-\cos 2\theta - i\sin 2\theta}\right)$$

$$= \text{Re}\left(\frac{2ie^{i\theta}}{2\sin^2\theta - i2\sin\theta\cos\theta}\right) = \text{Re}\frac{(2ie^{i\theta})i}{2\sin\theta(\sin\theta - i\cos\theta)i} = \text{Re}\left(-\frac{1}{\sin\theta}\right) = (-\infty, -1]\cup[1, \infty)$$

(B) $3^x = t$

$$-1 \le \frac{8t}{9-t^2} \le 1$$

$$0 \le \frac{t^2 - 8t - 9}{t^2 - 9}$$

$$0 \le \frac{(t-9)(t+1)}{(t-3)(t+3)}$$

$t < 3 \cup 9 \le t$

$x < 1 \cup 2 \le x$ …(i)

where $\frac{8t}{9-t^2} \le 1$

$$\frac{t^2 + 8t - 9}{(9-t^2)} \le 0$$

$$\frac{(t+9)(t-1)}{(3-t)(3+t)} \le 0$$

$t \le 1 \cup 3 < t$

$x \le 0 \cup 1 < x$ …(ii)

from (i) & (ii)

$x \le 0 \cup 2 \le x$

(C) $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$

$= 1(1+\tan^2\theta) - \tan\theta(-\tan\theta + \tan\theta) + 1(\tan^2\theta + 1) = 2\sec^2\theta$

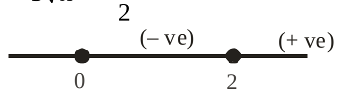
$f(\theta) \ge 2$

(D) $f(x) = x^{3/2}(3x - 10)$

$f'(x) = x^{3/2} \times 3 + (3x - 10)\frac{3}{2}x^{1/2}$

$$= 3\sqrt{x}\left(x + \frac{3x-10}{2}\right)$$

$= 3\sqrt{x}\,\frac{(5x-10)}{2}$



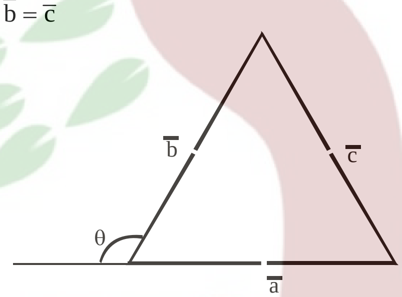
$x \geq 2$

60. Match the statement given in Column I with the values given in Column II

	Column I		Column II
(A)	If $\vec{a} = \hat{j}+\sqrt{3}\hat{k}$, $\vec{b} = -\hat{j}+\sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\,\hat{k}$ form a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is	(p)	$\frac{\pi}{6}$
(B)	If $\int_a^b (f(x)-3x)\,dx = a^2 - b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is	(q)	$\frac{2\pi}{3}$
(C)	The value of $\frac{\pi^2}{\ln 3}\int_{7/6}^{5/6} \sec \pi x)\,dx$ is	(r)	$\frac{\pi}{3}$
(D)	The maximum value of $\left\lvert \text{Arg}\left(\frac{1}{1-z}\right)\right\rvert$ for $\lvert z\rvert = 1$, $z \neq 1$ is given by	(s)	π
		(t)	$\frac{\pi}{2}$

Key: (A-q), (B-p,q,r,s,t), (C-s), (D-)

Sol.: (A) $\bar{a}+\bar{b}=\bar{c}$



$$\cos\theta = \frac{\bar{a}.\bar{b}}{|\bar{a}||\bar{b}|} = \frac{-1+3}{2\times 2} = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

Required angle $= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

(B) $\int_a^b f(x)dx = \frac{b^2-a^2}{2} \Rightarrow f\left(\frac{\pi}{6}\right) =$ can take many values

(C) $\frac{\pi^2}{\ln 3}\int_{7/6}^{5/6}(\sec \pi x)dx = \frac{\pi^2}{\ln 3}\frac{\left[\ln|\sec \pi x + \tan \pi x|\right]_{7/6}^{5/6}}{\pi}$

$$= \frac{\pi}{\ln 3}\left(\ln\left|\frac{3}{\sqrt{3}}\right| - \ln\left|\frac{1}{\sqrt{3}}\right|\right) = \pi$$

(D) $\left|\text{Arg}\left(\frac{1}{1-z}\right)\right| < \frac{\pi}{2}$

Hence maxima does not exist.